STEREO RIG GEOMETRY DETERMINATION BY FUNDAMENTAL MATRIX DECOMPOSITION

Carles Matabosch, Joaquim Salvi and Josep Forest

Universitat de Girona, Institut d'Informàtica i Aplicacions
17071 Girona, Catalonia (Spain)
email: {cmatabos, qsalvi, forest}@eia.udg.es

Abstract

Computer vision plays an important role in many automatic applications, such as surface measurement, industrial inspection, reverse engineering and even mobile robot navigation. All these applications might be solved by means of a stereovision system formed by at least two cameras if the geometry of such stereo rig is previously known. So, camera calibration is based on computing the intrinsic parameters, which determine the internal geometry and optics of the camera, and the extrinsic parameters, which determine the camera position and orientation with respect to a world coordinate system. In this paper, we present experimental results of computing the camera parameters without the knowledge of 3D information, which leads to the so-called self-calibration problem.

1 Introduction

Camera calibration is the first step towards computational computer vision. Some authors proposed many methods to obtain intrinsic camera parameters and the rigid movement between two cameras from the knowledge of a set of 3D points, which are considered the calibrating pattern (Salvi et al. 2002). However, depending on the visual application, the use of a calibrating pattern is not always available. In all these situations, a self-calibration method is recommended. Self-calibration is based on the properties of the projection on the image plane of the absolute conic. The absolute conic is a special conic that lies in a plane located in the infinity, having the property that its image projection depends on the intrinsic parameters only. This fact is expressed mathematically by the so-called Kruppa equations. The resolution of such equations leads to the computation of the intrinsic camera parameters, which are then used to get the extrinsic ones.

Hartley proposed a method that solved the Kruppa equations with a non-iterative algorithm. However, only a few number of all the intrinsic parameters are obtained, the others are considered previously known (Hartley 1992). Other authors expressed the same equations in a different form in order to extend the computation to the estimation of all the intrinsic parameters. For instance, Faugeras et al. used a method based on the estimation of the epipolar constraints of the image of the absolute conic (Faugeras et al. 1992). The same idea was described in (Luong & Faugeras 1993) where the authors calculated the rigid movement between both images up to a scale factor; and in (Zang et al. 1993) where even a projective reconstruction was given.

In all these methods, extreme accuracy is required in the segmentation of the 2D image points and the methods can hardly handle more than 4 views. In order to overcome this problem,

Hartley (Hartley 1993) proposed a technique that is suitable to a large numbers of views. Hartley proposed a Levenberg-Marquard algorithm to find the Projective reconstruction of a scene up to an unknown scale factor. A year latter, Hartley calculated the intrinsic parameters of a single camera taking several images from the same position, but rotating the camera (Hartley 1994). The author explained that it is possible to estimate intrinsic parameters with only two views, but some assumptions are required, i.e. orthogonal axis of the image plane ($\theta = \pi/2$) and square pixels ($\alpha_u = \alpha_v$). Besides, Song De Ma (De Ma 2000) used a method to compute the intrinsic parameters with six pure translations leading to a linear minimization. Mohr et al.(Mohr & Triggs 1996) and Hartley (Hartley 1997) introduced the SVD (Single Value Decomposition) of the fundamental matrix with the aim of simplifying the Kruppa equations. Moreover, he analysed the difficulty of finding a solution because of the complexity of the equations, i.e. the multivariable quadratics in the coefficients of the Kruppa matrix. Lourakis et al. (Lourakis & Deriche 1999) used simplified Kruppa equations considering the constraints of Huang-Faugeras and Trivedi to find a more robust solution against noise. These constraints are based on the general rank-2 constraint of the Essential matrix. Although Lourakis et al. (Lourakis & Deriche 2000) introduced a method to solve the case of varying intrinsic parameters, most of the proposed methods are based on constant intrinsic parameters.
Once the intrinsic parameters are known, it is possible to obtain the extrinsic ones by the decomposition of the Essential Matrix (Huang & Faugeras 1989). The Essential matrix represents the rotation and translation between both views and it can be easily computed from the Fundamental and both Intrinsic matrices.
In this paper, we discuss a known method to find the camera parameters by means of self-calibration. Section 2 describes the geometry of a pinhole camera and the given parameters. Then, section 3 explains the self-computation of the intrinsic parameters from the Kruppa equations. Moreover, section 4 deals with the computation of the extrinsic parameters from the Essential matrix. Finally, section 5 analyses the experimental results obtained from synthetic images under Gaussian noise. The paper ends with the conclusions.

2 The pinhole camera model

A model is a mathematical formulation that approximates the behaviour of any physical device by using a set of mathematical equations. Camera modelling is based on approximating the internal geometry and the position and orientation of the camera in the scene with a set of parameters. The pinhole model defines such relationship using equation (1).

$$s\mathbf{m} = \mathbf{A}\begin{bmatrix} \mathbf{R} & \mathbf{t} \\ \mathbf{0}_{1x3} & 1 \end{bmatrix} \mathrm{M} \qquad \mathbf{A} = \begin{bmatrix} \alpha_u & -\alpha_u \cot\theta & u_o & 0 \\ 0 & \alpha_v / \sin\theta & v_o & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{1}$$

where s is a nonzero scale factor, **m** are the coordinates of a point in a camera image in pixels, **M** are the metric coordinates of a 3D point, R and t represent respectively the rotation matrix and the translation vector defining the rigid movement between the camera and the world coordinate system. A is the matrix of the intrinsic parameters of the camera. The parameters α_u and α_v correspond to the focal distances in pixels along the axis of the image, θ (skew angle) is the angle between both image axis (usually near to $\pi/2$) and (u_0,v_0) are the coordinates of the projection of the focal point in the image plane (principal point).
Considering a pair of images of a given scene taking by two different cameras, points on both image planes are related by the epipolar geometry described by equation (2).

$$\mathbf{m}'^{t}\,\mathbf{F}\,\mathbf{m} = 0 \tag{2}$$

where F is known as the Fundamental matrix (Armangué & Salvi 2003) and m and m' are both projections of a given 3D point in the image planes. If only a single camera is used or many with the same intrinsic parameters, the Fundamental matrix is decomposed as follows,

$$\mathbf{F} = \mathbf{A}^{*}[\mathbf{t}]_{\mathbf{x}}\mathbf{R}\mathbf{A}^{-1} \tag{3}$$

where $\mathbf{A}^{*} = (\mathbf{A}^{-1})^{t}$ is the adjoint matrix of $\mathbf{A}$ and $[\mathbf{t}]_{x}$ denotes the antisymmetric matrix of the vector $\mathbf{t}$ that is associated with the cross product, and $\mathbf{R}$ is the rotation matrix between both cameras.

3 Kruppa equations

Several authors proposed different methods to decompose **F** to obtain the intrinsic and extrinsic parameters. Most of these methods are based on the Kruppa equations, which are obtained from a geometric interpretation of the absolute conic (AC). The absolute conic is a virtual conic that lies on a plane located in the infinity, a representation is shown in figure 1.

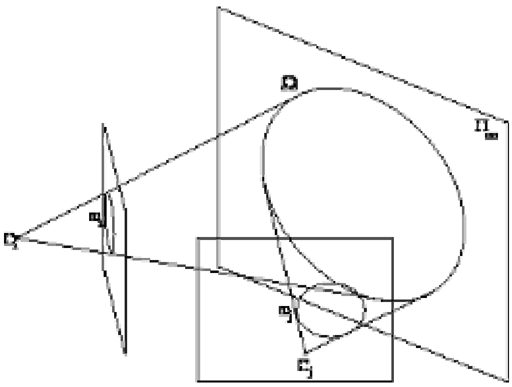

Figure 1 :The absolute conic (located in the plane at infinity) and its projection in both image planes.

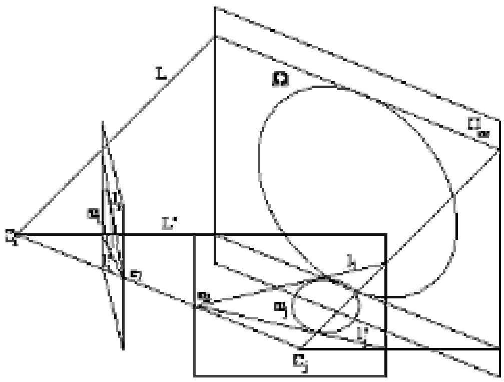

Figure 2. The Kruppa equations impose that the image of the absolute conic satisfies the epipolar constraint.

The image of the absolute conic (IAC) is invariant to rigid transformations so that its projection on the image plane do not depend on the position nor on the orientation of the camera. Therefore, IAC depends only on the intrinsic parameters of the camera. As can be seen in the figure 2, there exist two planes tangent to AC and containing the focal point of both cameras. These two planes intersect both image planes at two pairs of epipolar lines, which are tangent to IAC. The two IAC projected on both images are self-dual objects meaning that both IAC have the same coordinates in both camera planes, leading to equal projecting points and epipolar lines. Moreover, if we consider a single moving camera with fixed intrinsic parameters as a stereo rig system, the two intrinsic matrices, which define the IAC, are equal. Then, the property of the epipolar lines tangent to a dual conic is the following:

$$\mathbf{l}^{T}\,\mathbf{A}\mathbf{A}^{T}\,\mathbf{l} = 0 \tag{4}$$

As **l** is an epipolar line in any of both image planes (we have supposed the second one), it can be decomposed by:

$$(\mathbf{e}'\times\mathbf{m})^{t}\,\mathbf{A}\mathbf{A}^{t}\,(\mathbf{e}'\times\mathbf{m}) = 0 \tag{5}$$

Where **e'** is the epipol in the second camera in pixels. Moreover, the eipolar line $\mathbf{F}^t\mathbf{m}$ corresponding to **m** in the first image is also tangent to ω. Therefore, the invariance of ω under rigid transformations yields:

$$(\mathbf{F}^t\mathbf{m})^t\mathbf{A}\mathbf{A}^t(\mathbf{F}^t\mathbf{m}) = 0 \qquad (6)$$

Equation (5) and (6) can be transformed to:

$$\mathbf{m}([\mathbf{e}']_\times)^t\mathbf{A}\mathbf{A}^t([\mathbf{e}']_\times)\mathbf{m} = 0 \qquad (7)$$

and

$$\mathbf{m}^t\mathbf{F}\mathbf{A}\mathbf{A}^t\mathbf{F}^t\mathbf{m} = 0 \qquad (8)$$

Using equations (7) and (8), the following relationship is given:

$$\mathbf{F}\mathbf{A}\mathbf{A}^t\mathbf{F}^t = \beta\left(([\mathbf{e}']_\times)^t\mathbf{A}\mathbf{A}^t([\mathbf{e}']_\times)\right) \qquad (9)$$

Then, the product AA^T is expressed as a single matrix K,

$$\begin{pmatrix} k_1 & k_2 & k_3 \\ k_2 & k_4 & k_5 \\ k_3 & k_5 & 1 \end{pmatrix} = \begin{pmatrix} \alpha_u & 0 & u_0 \\ 0 & \alpha_v & v_0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \alpha_u & 0 & 0 \\ 0 & \alpha_v & 0 \\ u_0 & v_0 & 1 \end{pmatrix} = \begin{pmatrix} \alpha_u{}^2 + u_0{}^2 & u_0 v_0 & u_0 \\ u_0 v_0 & \alpha_u{}^2 + u_0{}^2 & v_0 \\ u_0 & v_0 & 1 \end{pmatrix} \qquad (10)$$

so that,

$$\mathbf{F}\mathbf{K}\mathbf{F}^t = \beta\left(([\mathbf{e}']_\times)^t\mathbf{K}([\mathbf{e}']_\times)\right) \qquad (11)$$

Both parts of equation (11) are defined by a 3x3 symmetric matrix. Therefore, every component on left must be equal to its homologue on the right. Then, the following 6 equalities are determined,

$$\frac{\mathbf{FKF}^t_{11}}{\left(([\mathbf{e}']_x)^t\mathbf{K}[\mathbf{e}']_x\right)_{11}} = \frac{\mathbf{FKF}^t_{12}}{\left(([\mathbf{e}']_x)^t\mathbf{K}[\mathbf{e}']_x\right)_{12}} = \frac{\mathbf{FKF}^t_{22}}{\left(([\mathbf{e}']_x)^t\mathbf{K}[\mathbf{e}']_x\right)_{22}} = $$
$$= \frac{\mathbf{FKF}^t_{13}}{\left(([\mathbf{e}']_x)^t\mathbf{K}[\mathbf{e}']_x\right)_{13}} = \frac{\mathbf{FKF}^t_{23}}{\left(([\mathbf{e}']_x)^t\mathbf{K}[\mathbf{e}']_x\right)_{23}} = \frac{\mathbf{FKF}^t_{33}}{\left(([\mathbf{e}']_x)^t\mathbf{K}[\mathbf{e}']_x\right)_{33}} \qquad (12)$$

Equations (12) are known as the Kruppa equations and express the geometrical constraint that the epipolar lines in both image planes are tangent to ω (see fig.2). However, equation (12) can be simplified with the aim of avoiding the use of the epipole **e'**, since its accurate estimation is difficult in the presence of noise. Furthermore, the simplified Kruppa equations are only three reducing the self-calibration problem. In order to extract the simplified Kruppa equations (Hartley 1997)(Lourakis & Deriche 1999), the Singular Value Decomposition (SVD) of the Fundamental matrix is employed:

$$\mathbf{F} = \mathbf{U}\mathbf{D}\mathbf{V}^{\mathbf{T}} \text{ where } \mathbf{D} = \begin{bmatrix} r & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad (13)$$

As **F** is of rank 2, the diagonal matrix **D** only have two non-null eigenvalues. Therefore, e' can be deduced from equation (14).

$$\mathbf{F}^t e' = \mathbf{V}\mathbf{D}^t\mathbf{U}^t e' = \mathbf{0} \qquad (14)$$

Using equation (14), the following solution for **e'** is obtained:

$$\mathbf{e}' = \delta\mathbf{U}\mathbf{m},\ \delta \neq 0 \qquad (15)$$

where $\mathbf{m}=[0,0,1]^t$ is the origin of the fisrt camera in the plane (principal point). Equation (15) can be validated by substituting **e'** in equation (14).

$$\mathbf{VD}^t\mathbf{U}^t\mathbf{Um} = \mathbf{V}\begin{bmatrix} r & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = V\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (16)$$

Therefore, equation (17) is satisfied.

$$[\mathbf{e}']_x = \mu\mathbf{UMU}^t \quad (17)$$

where μ is an nonzero scale factor and $\mathbf{M}=[\mathbf{m}]_x$.
By substituting equation (17) into equation (12), a new expression for the Kruppa equations is obtained:

$$\mathbf{FKF}^t = \mu\mathbf{UMU}^t\mathbf{KUM}^t\mathbf{U}^t \quad (18)$$

Since $\mathbf{U}$ is an orthogonal matrix, left and right multiplication of equation (18) by $\mathbf{U}^t$ and $\mathbf{U}$ respectively, yields to the simplified expression of the Kruppa equations:

$$\mathbf{DV}^t\mathbf{KVD}^t = \mu\mathbf{MU}^t\mathbf{KUM}^t \quad (19)$$

Operating these matrix, we obtain the following equation

$$\begin{bmatrix} r^2\mathbf{v_1}^t\mathbf{Kv_1} & rs\mathbf{v_1}^t\mathbf{Kv_2} & 0 \\ rs\mathbf{v_2}^t\mathbf{Kv_1} & s^2\mathbf{v_2}^t\mathbf{Kv_2} & 0 \\ 0 & 0 & 0 \end{bmatrix} = \mu\begin{bmatrix} \mathbf{u_2}^t\mathbf{Ku_2} & -\mathbf{u_2}^t\mathbf{Ku_1} & 0 \\ -\mathbf{u_1}^t\mathbf{Ku_2} & \mathbf{u_1}^t\mathbf{Ku_1} & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (20)$$

In order to satisfy equation (20), every component on the left must be equal to its homologue on the right, obtaining equation (21).

$$\frac{r^2\mathbf{v}_1^t\mathbf{Kv}_1}{\mathbf{u}_2^t\mathbf{Ku}_2} = \frac{rs\mathbf{v}_1^t\mathbf{Kv}_2}{-\mathbf{u}_2^t\mathbf{Ku}_1} = \frac{s^2\mathbf{v}_2^t\mathbf{Kv}_2}{\mathbf{u}_1^t\mathbf{Ku}_1} \quad (21)$$

These equations were first presented by Hartley. However, the complexity of finding the proper solution caused by the existence of multiple solutions in a non-linear system and the complexity of solving redundant systems, leads to the lack of experimental results (Hartley 1997). Then, Hartley assumed that there is no skew parameter (that means $\theta = \pi/2$) and the image of the principal point is in the image centre. In this case, both images lead to a pair of quadratic equations with two unknowns, which can be solved as explained in the following paragraphs.
First, equation (21) is represented as equation (22).

$$\frac{\rho_1}{\varphi_1} = \frac{\rho_2}{\varphi_2} = \frac{\rho_3}{\varphi_3} \quad (22)$$

Then, an approximation of $\mathbf{K}$ is done to find an initial solution, then an iterative non-linear method is used to minimize to the global solution.

4.1 Initial solution by Hartley

In order to find an initial solution, the skew angle θ is assumed equal to $\pi/2$, and the position of the principal point in the image is assumed to coincide with the centre of the image. Using both simplifications, the number of unknowns is reduced to two, namely k_1 and k_4, which are related to the focal lengths α_u and α_v. Then, an initial solution is obtained solving equation (23).

$$\begin{aligned} \rho_1\varphi_2 - \varphi_1\rho_2 &= 0 \\ \rho_1\varphi_3 - \varphi_1\rho_3 &= 0 \end{aligned} \quad (23)$$

However, the system is of second order, which doesn't have an implicit solution. Then, the initial solution is found numerically.

4.2 Minimization criteria by Lourakis

Once an initial solution is found, it is possible to converge to a more accurate solution minimizing the following function.

$$\sum_{i=1}^{N} \frac{\pi_{12}^2}{\sigma_{\pi_{12}}^2} + \frac{\pi_{13}^2}{\sigma_{\pi_{13}}^2} + \frac{\pi_{23}^2}{\sigma_{\pi_{23}}^2} \tag{24}$$

where $\pi_{ij} = \frac{\rho_i}{\phi_i} - \frac{\rho_j}{\phi_j}$, and $\sigma_{\pi_{ij}}^2$ is the variance, which can be approximated by

$$\sigma_{\pi_{ij}}^2 = \frac{\partial \pi_{ij}}{\partial S_F} \Lambda_{S_F} \frac{\partial \pi_{ij}}{\partial S_F}^t \tag{25}$$

Where $S_F = [r,s,u_1^t, u_2^t, u_3^t, v_1^t, v_2^t, v_3^t]^t$, a 20×1 vector composed by the parameters of the SVD of **F**. $\frac{\partial \pi_{ij}}{\partial S_F}$ is the derivative of π_{ij} at S_F, and Λ_{S_F} is the covariance matrix.

Although a classical Levenberg-Marquardt algorithm can be used in the minimization, we have obtained a proper convergence of the method using the criteria proposed by Triggs and explained in the following paragraphs.

4.3 Minimization criteria by Triggs

The method proposed by Triggs (Triggs 1997) is a general numerical scheme for optimizing smooth non-linear cost functions under smooth non-linear constraints. The goal is to minimize a scalar cost function *f(x)* subject to a vector of constraints $\mathbf{c}(x) = 0$.

$$\nabla f + z \cdot \nabla \mathbf{c} = \mathbf{0} \quad \text{with} \quad \mathbf{c}(x) = 0 \tag{26}$$

An exact linear solution can be solved with the following equation

$$\begin{pmatrix} \nabla^2 f & \nabla \mathbf{c}^T \\ \nabla \mathbf{c} & 0 \end{pmatrix} \begin{pmatrix} \boldsymbol{\delta} \mathbf{x} \\ z \end{pmatrix} = -\begin{pmatrix} \nabla f \\ \mathbf{c} \end{pmatrix} \tag{27}$$

where we have to solve for $\delta\mathbf{x}$ from the initial solution $\mathbf{x_0}$ and then compute $\mathbf{x_1} = \mathbf{x_0} + \delta\mathbf{x}$, compute again the derivatives at $\mathbf{x_1}$, and iterate to convergence.

Equation (23) is derivate by k_1, k_2, k_3, k_4 and k_5 in order to obtain ∇, and again to obtain ∇^2. In our application, three images are used in order to find three pairs of simplified Kruppa equations like the ones shown in equation (23). Then f is one of these six equations, and any four of the other five equations are used as constraints. Note that always one of the six equations is a linear combination of the others. Furthermore, another constraint, see equation (28) and equation (11), is used in order to improve the robustness of the minimization against noise.

$$k_2 - k_3 k_5 = 0 \tag{28}$$

If the Gaussian noise is null, any combinations of the six equations (see equation (23)) leads to the correct solution. However, in the presence of Gaussian noise, some combinations lead to better results than others. Then, the proper way is to solve the system for all the combinations and then choose the one that the minimize the constraint of Huang-Faugeras shown in equation (29).

$$\text{trace}^2\left(\mathbf{A}^t\mathbf{F}\mathbf{A}\mathbf{A}^t\mathbf{F}^t\mathbf{A}\right) - 2\text{trace}\left(\left(\mathbf{A}^t\mathbf{F}\mathbf{A}\mathbf{A}^t\mathbf{F}^t\mathbf{A}\right)^2\right) = 0 \tag{29}$$

Equation (29) determines that the essential matrix have to be rank two.

4. Determination of the rigid movement

The determination of the rigid movement is given by computing the rotation matrix and translation vector between both cameras, which can be obtained by the decomposition of the fundamental matrix of equation (30)

$$\mathbf{F}=\mathbf{A}^{*}\mathbf{E}\mathbf{A}^{-1} \tag{30}$$

where $\mathbf{E} = [\mathbf{t}]_x\mathbf{R}$. Then, once the intrinsic parameters are determined using the algorithms detailed in section 3, the extrinsic parameters can be extracted as using equation (31) or (32).

$$\mathbf{R} = \mathbf{U}\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\mathbf{V}^{\mathbf{T}} \qquad [\mathbf{t}]_x = \mathbf{E}(\mathbf{R})^{-1} \tag{31}$$

or

$$\mathbf{R} = \mathbf{U}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\mathbf{V}^{\mathbf{T}} \qquad [\mathbf{t}]_x = \mathbf{E}(\mathbf{R})^{-1} \tag{32}$$

where $\mathbf{U}$ and $\mathbf{V}$ are obtained by decomposition of $\mathbf{E}=\mathbf{U}\mathbf{D}\mathbf{V}^{T}$. The right solution is the one that locates the 3D points in front of both cameras.

5. Experiments results

The algorithm explained in section 3 and 4 has been programmed in Matlab. Experimental results are obtained from a set of 25 synthetic points, which defines a 3D scene. Three different views of the 3D scene are taken by using a single moving camera. The camera has been modelled as a pinhole camera and the intrinsic and extrinsic parameters are fixed in order to compute the 2D projections in pixels of the 3D scene. Then, three fundamental matrices are calculated by means of eigen analysis from the three projections.

These fundamental matrices are used to estimate the intrinsic parameters using the method discussed in section 3. In order to observe the robustness of this method, Gaussian noise were introduced in the pixel coordinates. Figure 3 shows the robustness of the proposed method in the presence of such noise.

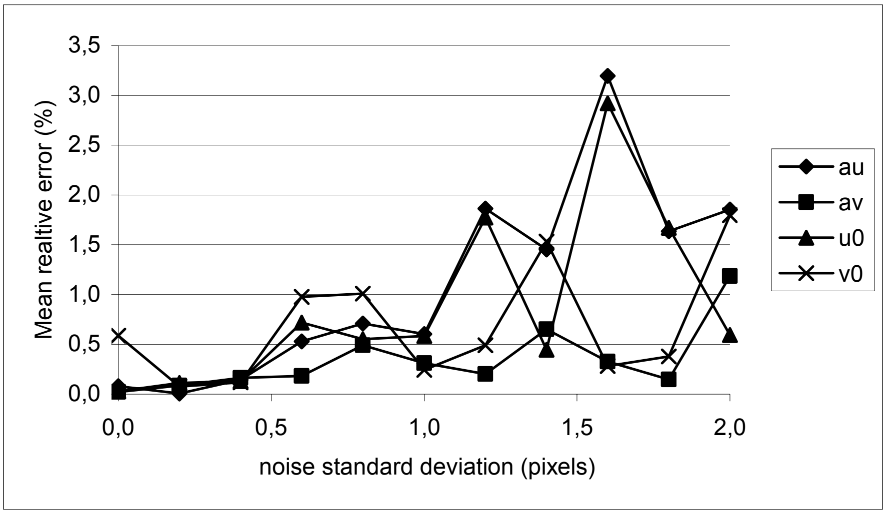

Figure 3: The error in the intrinsic parameters estimation in the presence of noise

6 Conclusions

Camera calibration is a requisite and a crucial problem in a lot of vision task. In some applications, calibration can be determinated off-line as a previous step in the vision task. However, in other applications this step is not always possible. Then, self-calibration methods are used in order to calibrate the camera using the same views acquired by the vision system on-line. In this paper, a method to resolve this problem is described. The method is based in the constraints of the image of the absolute conic, which are expressed in the so-called Kruppa equations. Since the obtained equation system is non-linear, iterative methods are used in minimization. Furthermore, some constraints are introduced in the minimization process with the aim of converging to the proper solution. Once the intrinsic parameters are known from minimization, a method to find the extrinsic parameters is shown in order to find the rigid movement between both cameras. Then, a projective reconstruction can be generated by using the set of intrinsic and extrinsic parameters. Experimental results are given which show the robustness of the method in the presence of Gaussian noise. Moreover, the whole algorithm is available in the webpage of the authors.

References

Armangué, X. & Salvi, J. (2003), *Overall View Regarding Fundamental Matrix Estimation*. Image and Vision Computing **21**(2), pp 205-220,

Faugeras, O.D. Luong, Q.T..& Maybank, S.J. (1992), *Camera Self-Calibration: Theory and Experiments*. In Computer Vision -ECCV '92, **588**, pp 321-334.

Hartley, R.I.(1992) *Estimation of relative camera positions for uncalibrated cameras*. Proceedings of the 2nd European Conference on Computer Vision, pages 579-587.

Hartley, H.I. (1993) *Euclidean Reconstruction from Uncalibrated views*. In Proc. On the Second Europe-US Workshop on Invariance, pp187-202.

Hartley, H.I. (1994), *Self-calibration from multiple views with a rotating camera*. In J-O. Eklundh, editor, Proceedings of the 3rd European Conference on Computer Vision, **800-801** of Lecture Notes in Computer Science, pp 471-478, Stockholm

Hartley,R.I.(1997) *Kruppa's Equations Derived from the Fundamental Matrix*. IEEE Transactions on Pattern Analysis and Machine Intelligence, **19**(2):133-135.

Huang, T.S. & Faugeras, O.D.(1989), *Some Properties of the E Matrix in Two-View Motion Estimation*. IEEE Tran.Pattern Analysis on Machine Intelligence, **11**(12):1310-1312.

Luong, Q.T. & Faugeras O.D. (1993), *Self-calibration of a stereo rig from unknown camera motions and point correspondences*. Research Report **2014**, INRIA Sophia-Antipolis

Lourakis, M.I.A & Deriche, R. (1999), *Camera Self-Calibration using the Singular Value Decomposition of the Fundamental Matrix: From Point Correspondences to 3D Measurements*. Research Report **3748**, INRIA Sophia-Antipolis, August 1999

Lourakis, M.I.A & Deriche, R. (2000), *Camera Self-Calibration using the Singular Value Decomposition of the Fundamental Matrix: The case of varying intrinsic parameters.* Research Report **3911**, INRIA Sophia-Antipolis.

De Ma.S.(1996) *A self-Calibration Technique for Active Vision Systems*. IEEE Transactions on Robotics and Automation, Vol **12**. nº1. pp 114-120.

Mohr, R. & Triggs, B.(1996), *Projective geometry for image analysis*. ISPRS workshop tutorial, Vienna, Austria.

Salvi, J., Armangué, X., & Batlle, J.(2002) *A Comparative Review of Camera Calibrating Methods with Accuracy Evaluation*. Pattern Recognition **35**(7), pp 1617-1635.

Triggs,B.(1997) *Autocalibration and the Absolute Quadric*. In IEEE International Conference on Computer Vision and Pattern Recognition, pages 609-614.

Zhang, Z., Luong, Q.T. & Faugeras, O. (1993), *Motion of an uncalibrated stereo rig:Self-calibration and metric reconstruction*.Research Report**2079**,INRIA Sophia-Antipolis