

# Formulari de Robòtica

$$\begin{pmatrix} {}^A p_1 \\ {}^A p_2 \\ {}^A p_3 \end{pmatrix} = \begin{pmatrix} a_1 \cdot b_1 & a_1 \cdot b_2 & a_1 \cdot b_3 \\ a_2 \cdot b_1 & a_2 \cdot b_2 & a_2 \cdot b_3 \\ a_3 \cdot b_1 & a_3 \cdot b_2 & a_3 \cdot b_3 \end{pmatrix} \begin{pmatrix} {}^B p_1 \\ {}^B p_2 \\ {}^B p_3 \end{pmatrix}$$

{A} sistema de coordenades amb vectors unitaris  $a_1, a_2, a_3$

{B} sistema de coordenades amb vectors unitaris  $b_1, b_2, b_3$

$$\text{Rot}(x, \mathbf{q}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \mathbf{q} & -\sin \mathbf{q} & 0 \\ 0 & \sin \mathbf{q} & \cos \mathbf{q} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{Rot}(y, \mathbf{a}) = \begin{pmatrix} \cos \mathbf{a} & 0 & \sin \mathbf{a} & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \mathbf{a} & 0 & \cos \mathbf{a} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\text{Rot}(z, \mathbf{y}) = \begin{pmatrix} \cos \mathbf{y} & -\sin \mathbf{y} & 0 & 0 \\ \sin \mathbf{y} & \cos \mathbf{y} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{Trans}(o_x, o_y, o_z) = \begin{pmatrix} 1 & 0 & 0 & o_x \\ 0 & 1 & 0 & o_y \\ 0 & 0 & 1 & o_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$T = \left( \begin{array}{ccc|c} R & & & p \\ \hline 0 & 0 & 0 & 1 \end{array} \right) \Rightarrow T^{-1} = \left( \begin{array}{ccc|c} R^T & & & -R^T \cdot p \\ \hline 0 & 0 & 0 & 1 \end{array} \right)$$

$${}^{k-1}A_k = \begin{pmatrix} Cq_k & -Ca_k Sq_k & Sa_k Sq_k & a_k Cq_k \\ Sq_k & Ca_k Cq_k & -Sa_k Cq_k & a_k Sq_k \\ 0 & Sa_k & Ca_k & d_k \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

## Interpolador Lineal

$$q(t) = (q^i - q^{i-1}) \cdot \frac{t - t^{i-1}}{T} + q^{i-1} \quad \text{per} \quad t^{i-1} < t < t^i$$

$$T = t^i - t^{i-1}$$

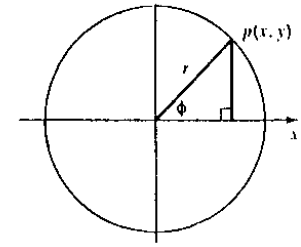
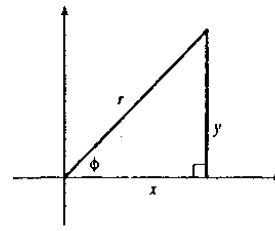


Figure A.1

$$\sin(\phi) = y/r$$

$$\cos(\phi) = x/r$$

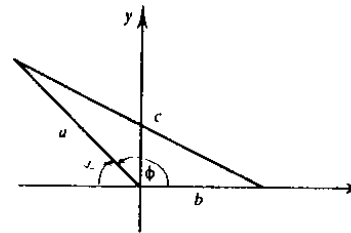
$$\tan(\phi) = y/x = \sin(\phi)/\cos(\phi)$$

$$x^2 + y^2 = r^2 \quad \text{Pythagoras' Theorem}$$

$$\cos(\phi) = \sin(90 - \phi)$$

arc = radius  $\times$   $\phi$ , where  $\phi$  is in radians, and 1 radian =  $57^\circ 18'$

$\pi$  radians in  $180^\circ$ , where  $\pi = 3.14159$



$$c^2 = a^2 + b^2 - 2ab\cos(\phi) \quad \cong \quad c^2 = a^2 + b^2 + 2ab\cos(\alpha)$$

$$\sin(-\phi) = -\sin(\phi) \quad \cos(-\phi) = \cos(\phi)$$

$$\cos^2(\phi) + \sin^2(\phi) = 1 \quad \cos(2\phi) = \cos^2(\phi) - \sin^2(\phi)$$

$$\tan^2(\phi) + 1 = \sec^2(\phi) \quad \sin(2\phi) = 2\cos(\phi)\sin(\phi)$$

$$\sin(\theta + \phi) = \sin(\theta)\cos(\phi) + \cos(\theta)\sin(\phi)$$

$$\cos(\theta + \phi) = \cos(\theta)\cos(\phi) - \sin(\theta)\sin(\phi)$$

$$\sin(\theta - \phi) = \sin(\theta)\cos(\phi) - \cos(\theta)\sin(\phi)$$

$$\cos(\theta - \phi) = \cos(\theta)\cos(\phi) + \sin(\theta)\sin(\phi)$$

$$\tan(2\phi) = \frac{2\tan(\phi)}{1 - \tan^2(\phi)}$$

$$\tan(\phi - \theta) = \frac{\tan(\phi) - \tan(\theta)}{1 + \tan(\phi)\tan(\theta)}$$

$$\tan(\phi + \theta) = \frac{\tan(\phi) + \tan(\theta)}{1 - \tan(\phi)\tan(\theta)}$$

If  $\alpha = \phi + \theta$ , and  $\beta = \phi - \theta$  then:

$$\sin(\alpha) - \sin(\beta) = 2\cos(\phi)\sin(\theta) = 2\cos\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)$$

# Algorisme de Denavit-Hartenberg (DH)

0. Numereu les **unions** des de 1 fins a  $n$  començant per la base i acabant pel yaw, pitch i roll de l'element final (en aquest ordre).
1. Assigneu el sistema de coordenades  $L_0$  a la base, assegurant-vos que  $Z_0$  coincideix amb l'eix de la unió 1, i inicialitzeu  $k=1$ .
2. Feu coincidir  $Z_k$  amb l'eix de la unió  $k+1$ .
3. Poseu l'origen de  $L_k$  a la intersecció dels eixos  $Z_k$  i  $Z_{k-1}$ . Si  $Z_k$  i  $Z_{k-1}$  no interseccen, utilitzeu la intersecció de  $Z_k$  amb una normal comuna a  $Z_k$  i  $Z_{k-1}$ .
4. Poseu  $X_k$  de manera que sigui ortogonal a  $Z_k$  i  $Z_{k-1}$ . Si  $Z_k$  i  $Z_{k-1}$  són paral·lels, feu  $X_k$  perpendicular a  $Z_{k-1}$ , situeu-lo al llarg del link i apuntant cap a fora.
5. Seleccioneu  $Y_k$  per acabar de completar el sistema de coordenades  $L_k$ .
6. Feu  $k=k+1$ . Si  $k < n$ , torneu al pas 2.
7. Poseu l'origen de  $L_n$  a la punta de l'element final. Feu coincidir  $Z_n$  amb el vector **a** (approach),  $Y_n$  amb el vector **o** (orientation) i  $X_n$  amb el vector normal de l'element final. Feu  $k=1$ .
8. Poseu el punt  $b_k$  a la intersecció dels eixos  $X_k$  i  $Z_{k-1}$ . Si no interseccen, poseu-lo a la intersecció d' $X_k$  amb una normal comuna a  $X_k$  i  $Z_k$ .
9.  $q_k$  és l'angle de rotació des de  $X_{k-1}$  a  $X_k$  mesurat sobre  $Z_{k-1}$ .
10.  $d_k$  és la distància des de l'origen del sistema de coordenades  $L_{k-1}$  al punt  $b_k$  mesurat al llarg de l'eix  $Z_{k-1}$ .
11.  $a_k$  és la distància des del punt  $b_k$  a l'origen del sistema de coordenades  $L_k$  mesurat al llarg de  $X_k$ .
12.  $\alpha_k$  és l'angle de rotació des de  $Z_{k-1}$  a  $Z_k$  mesurat sobre  $X_k$ .
13. Feu  $k=k+1$ . Si  $k \leq n$  aneu al pas 8.

Program yes ..... Possible  
 no ..... Not possible

Name	Input Format	Function	Program	Remarks
1 Decrease Position	DP	Moves robot to a predefined position with a position number smaller than the current one.	yes	
2 Draw	DW x, y, z	Moves hand end to a position away from the current one covering the distance specified in X, Y, and Z-axis directions.	yes	
3 Home	HE a	Defines the coordinates of the position number (a) to it.	yes	1 ≤ a ≤ 629
4 Home	HO	Finishes the reference position in the cartesian coordinate system.	yes	
5 Increment Position	IP	Moves robot to a predefined position with a position number greater than the current one.	yes	
6 Move Approach	MA a <sub>1</sub> , a <sub>2</sub>   O/C	Moves hand end from the current position to a position away from position (a <sub>1</sub> ) in increments as specified for position (a <sub>2</sub> ).	yes	1 ≤ a <sub>1</sub> , a <sub>2</sub> ≤ 629 O: Hand closed; C: Hand closed
7 Move Continuous	MC a <sub>1</sub> , a <sub>2</sub>	Moves robot continuously through predefined intermediate points between position numbers (a <sub>1</sub> ) and (a <sub>2</sub> ).	yes	1 ≤ a <sub>1</sub> , a <sub>2</sub> ≤ 629
8 Move Joint	MJ w, s, e, p, r	Turns each joint the specified angle from the current position.	no	
9 Move	MO a   O/C	Moves hand end to position (a).	yes	1 ≤ a ≤ 629 O: Hand opened; C: Hand closed
10 Move Position	MP x, y, z, p, r	Moves hand end to a position whose coordinates (position and angle) are specified as x, y, z, p, and r.	no	
11 Move Straight	MS a, n   O/C	Moves robot to position (a) through n intermediate points on a straight line.	yes	1 ≤ a ≤ 629 1 ≤ n ≤ 99 O: Hand opened; C: Hand closed
12 Move Tool	MT a, b   O/C	Moves hand end from the current position to a position away from a specified position (a) in incremental distance b in the tool direction.	yes	1 ≤ a ≤ 629 O: Hand opened; C: Hand closed
13 Nest	NT	Returns robot to mechanical origin.	yes	
14 Origin	OG	Moves robot to the reference position in the cartesian coordinate system.	yes	
15 Pallet Assign	PA i, j, k	Defines the number of grid points (j, k) in the column and row directions for pallet (i).	yes	1 ≤ i ≤ 9 1 ≤ j, k ≤ 255
16 Position Clear	PC a <sub>1</sub>   a <sub>2</sub>	Clears all position data from position a <sub>1</sub> to a <sub>2</sub> .	no	a <sub>1</sub> ≤ a <sub>2</sub> 1 ≤ a <sub>1</sub> , a <sub>2</sub> ≤ 629 (for a <sub>1</sub> = C)
17 Position Define	PD a, x, y, z, p, r	Defines the coordinates (x, y, z, p, r) of position (a).	no	1 ≤ a ≤ 629

Name	Input Format	Function	Program	Remarks
35	If Larger LG a <sub>1</sub> (or &b), a <sub>2</sub>	Causes a jump to occur to line number (a <sub>2</sub> ) if external input data or counter data is greater than a <sub>1</sub> (or &b).	Yes	(-32767) (32767) 0 ≤ a ≤ 255 (decimal) 0 ≤ b ≤ &FF (hex.) (&8001) (&7FFF) 1 ≤ a <sub>2</sub> ≤ 2048
36	If Not Equal NE a <sub>1</sub> (or &b), a <sub>2</sub>	Causes a jump to occur to line number (a <sub>2</sub> ) if external input data or counter data does not equal a <sub>1</sub> (or &b).	Yes	(-32767) (32767) 0 ≤ a ≤ 255 (decimal) 0 ≤ b ≤ &FF (hex.) (&8001) (&7FFF) 1 ≤ a <sub>2</sub> ≤ 2048
37	New NW	Deletes all program and position data in RAM.	No	
38	Next NX	Specifies the range of a loop in a program, executed by command RC	Yes	
39	Repeat Cycle RC a	Repeats the loop specified by command NX (a) times.	Yes	1 ≤ a ≤ 32767
40	Run RV a <sub>1</sub>   a <sub>2</sub>	Executes line numbers from (a <sub>1</sub> ) to (a <sub>2</sub> ) (a <sub>2</sub> ) not included.	No	1 ≤ a <sub>1</sub> , a <sub>2</sub> ≤ 2048
52	Return RT	Completes subroutines activated by command GS and returns to main program.	Yes	
42	Set Counter SC a <sub>1</sub>   a <sub>2</sub>	Loads (a <sub>2</sub> ) into counter (a <sub>1</sub> ).	Yes	1 ≤ a <sub>1</sub> ≤ 99 -32767 ≤ a <sub>2</sub> ≤ 32767
43	If Smaller SM a <sub>1</sub> (or &b), a <sub>2</sub>	Causes a jump to occur to line number (a <sub>2</sub> ) if external input data or counter data is smaller than a <sub>1</sub> (or &b).	Yes	(-32767) (32767) 0 ≤ a <sub>1</sub> ≤ 255 (decimal) 0 ≤ b ≤ &FF (hex.) (&8001) (&7FFF) 1 ≤ a <sub>2</sub> ≤ 2048

### C Hand Control Instructions

Name	Input Format	Function	Program	Remarks
44	Grip Close GC	Closes hand grip.	Yes	
45	Grip Flag GF a	Defines the open/close state of hand grip, used in conjunction with command PD.	Yes	a = 0 (open), 1 (closed)
46	Grip Open GO	Opens hand grip.	Yes	
47	Grip Pressure GP a <sub>1</sub> , a <sub>2</sub> , a <sub>3</sub>	Defines gripping force and gripping force retention time	Yes	0 ≤ a <sub>1</sub> , a <sub>2</sub> ≤ 15 0 ≤ a <sub>3</sub> ≤ 99 (Unit: 0.1 seconds)

### D VO Control Instructions

Name	Input Format	Function	Program	Remarks
48	Input Direct ID	Fetches external signal unconditionally from input port	Yes	
49	Input IN	Fetches external signal synchronously from input port.	Yes	