How-to demonstrate Incentive Compatibility in Multi-Attribute Auctions

Albert PLA[a] [1], Beatriz LÓPEZ[a] and Javier MURILLO[b]
[a] *Universitat de Girona, Girona, Spain* [b] *Newronia, Girona, Spain*
{*albert.pla,beatriz.lopez*}*@udg.edu javier.murillo@newronia.com*

Abstract. Demonstrating the incentive compatibility of an auction mechanism is always a hard but essential work in auction mechanism design. In this paper we discuss three different approaches to proof or check such property in regard of a multi-attribute auction mechanism: by analyzing well-known sufficient conditions, by mathematical analyzing the rules that govern the mechanism, and by empirically checking the mechanism. Particularly, for dealing with the second approach, we propose a new method which consists on seeking for a counterexample with a constraint solver.

Keywords. Multi-agent systems, Multi-attribute auctions, SMT Solvers

1. Introduction

In auction design, a mechanism is said to be incentive-compatible or strategyproof if all of the participants maximize their utility when they are bidding truthfully, revealing their private information and true values on the bid [1]. This reduces the chances of a bidder trying to manipulate market prices as bidders obtain their maximum benefit when they bid honestly. Proving incentive compatibility in auctions is always laborious.

There are three main ways of demonstrating that a mechanism fulfills such property. First, by proving that the mechanism satisfies certain sufficient conditions which make truthful bidding the dominant strategy (the strategy which maximizes its revenue if assuming that the rest of bidders are acting rationally) for bidders. Second, by formulating the rules that are used to determine the winners and to compute the payments, and mathematically deriving the demonstration. And third, incentive compatibility can be also derived from empirical experimentation, analyzing the revenue and utilities obtained by bidders who bid honestly and comparing them with those obtained by bidders who bid following other strategies. These are alternative methods. The sufficient conditions method seems to be the simplest one, but it is not always neither easy nor intuitive to achieve. Being successful on the second methodology hardly depends on the complexity of the mechanism. When this occurs, the third methods is the single alternative.

In this work we explore the usability of the methods in the particular case of the multi-attribute mechanism described in [2]. In that doing, we introduce as a first time the use of a satisfiability modulo theories solver (SMT solver) in the second method, saving a lot of time to researchers.

[1] Corresponding Author: Albert Pla, University of Girona, Campus Montilivi, P4 Building, 17071 Girona, Spain.

2. The Mechanism: Multi-attribute Auction

We are interested in proving the mechanism defined in [2]: a reverse second price multi-attribute auction mechanism used to allocate tasks to resource providers which bases its payment in the commitment of the agreements made during the auction. In a multi-attribute auction each bid B_i is characterized by a set of attributes, at_i, in addition to a price b_i, thus $B_i = (at_i, b_i)$ [3]. The winner determination problem consist on finding the bid which best satisfies the auctioneer regarding the price but also the rest of attributes according to the auctioneers score function, $V(at_i, b_i)$. In the particular case of our mechanism, first, attributes are combined by using an aggregation function $f(at_i) \in \Re$ [4], before the score function is used $V(f(at_i), b_i)$. The mechanism deals with reverse auctions, and therefore, the goal of the auctioneer is to minimize the economic cost (b_i) and the attributes (e.g. to minimize the duration of a task), $mimimize_i(V(f(at_i), b_i))$. Note that $f(at_i)$ should decrease when the values of at_i are better. The mechanism proposes three different evaluation functions: sum, the weighted sum and the product.

The payment mechanism is based in the classical Vickrey auction (where the winner pays the amount offered in the second best bid) but taking into account attributes as well as price. When the delivered attributes are the ones which were bid, payment is done following a second price philosophy. Otherwise, when the bidder delivers a set of attributes worse than those it had tendered, payment p corresponds to the amount the bidder should have bid in order to win the auction but with the real delivered attributes.

3. Demonstrating Incentive Compatibility

In this section we discuss the incentive compatibility of the previous mechanism when assuming that there are no externalities[2] following three different ways of demonstrations: analyzing of the mechanism properties, trying to find a counter example, and experimentally checking the mechanism.

3.1. Meeting sufficient conditions

For an auction mechanism to be incentive compatible it is sufficient to fulfill certain conditions [5]:

- **Exactness** postulates that a single minded bidder receives exactly the set of goods it desires or nothing. In the presented mechanism the auctioned item is assigned to only one winner, moreover, the only way of winning the auctioned item is to participate in the auction offering the best bid. Thus, bidders can not obtain items for which they have not bid: they get exactly what they bid for or nothing.
- **Participation** requests that unsatisfied bidders pay zero and their utility is zero. In the presented mechanism there is no fee to access the auction. Thus, if an agent does not win, it does not pay anything, obtaining 0 utility.
- **Monotonicity** requires that if a bidder increases its bid (decreases in reverse auctions) the bidder still wins the auction. This property is strictly related to the evaluation function. When using a monotonic function as evaluation function (e.g. the sum, the product or the weighted sum) the act of improving any of the attributes causes the bidder to gain a higher evaluation (decrease in reverse auctions). In consequence, improving a winning bid cannot cause a bidder to lose the auction.

[2] Bidders final payoff are determined solely by whether or not they obtain the auctioneed good and their payment.

- **Criticality** claims that each winning bidder pays (receives in reverse auctions) the lowest value it could have declared and still be allocated the good it requested. The payment mechanism computes the payment p by matching the evaluation of the second best bid with the evaluation of the payment and the attributes of the winning bid ($V(f(at_2), b_2) = V(f(at_1), p)$). This ensures that the winning bidder will receive the amount it should have tendered to obtain the same evaluation as the second best bid, ensuring that the payment is the minimum amount need to bid to win the auction with the bided attributes, thereby respecting criticality.

Since the mechanism commits the four mentioned above properties, we can deduce that the mechanism is incentive compatible if assuming no externalities.

3.2. Seeking for a counterexample with a constraint solver

To prove that truthful bidding is the dominant strategy we have to prove that, for any feasible bid, the utility of a bidder is higher or equal when bidding truthfully than when providing false attributes ($f(at_i) \neq f(at_i^v)$) or an economic bid different from the true value ($b_i^v \neq b_i$): Thereby, if we model the auction mechanism as a constraint satisfaction problem we can try to find a a case where the utility of a bidder would have been higher when lying than when telling the truth. If this case exists, then the mechanism is not incentive compatible. Thus, our goal is to find out whether these cases exist or not.

The auction mechanism and the counterexample can be modeled as an inequation system. If a SMT constraint solver is able to find a solution for the inequation system it will show that exists, at least, one case in which the utility of the bidder is higher when lying than when bidding honestly, refuting the hypothesis that the mechanism is incentive compatible. It is important to take into account the kind of functions which defines the auctioneer utilities, the winner determination problem and the payment rule as this will condition if the solver must support linear or non-linear arithmetics.

Assuming that the bidder utility function is $p - b_i^v$ when it wins and 0 when it does not, we can obtain the following inequation system when the product (Equation 1) is used as the score function V:

$$\left.\begin{array}{ll}
\text{(a)} & f(at_1) \neq f(at_1^v) \vee b_1 \neq b_1^v \\
\text{(b)} & b_1 * f(at_1) < b_2 * f(at_2) \\
\text{(c)} & win = \begin{cases} 1 \text{ if } (b_2 * f(at_2) > b_1^v * f(at_1^v)) \\ 0 \text{ otherwise} \end{cases} \\
 & \\
\text{(d)} & win * (\frac{b_2 * f(at_2)}{f(at_1^v)} - b_1^v) < (\frac{b_1 * f(at_1)}{f(at_1^v)} - b_1^v)
\end{array}\right\} \qquad (1)$$

where Eq.1a defines that bidder 1 is lying about at least one of the attributes it bids, Eq.1b defines that bidder 1 wins the auction, *win* in Eq.1c defines if the bidder would have won the auction by bidding truthfully and Eq.1d compares the utility obtained with the utility it would have obtained by bidding truthfully.

To test the satisfiability of the inequation system 1 we used Microsoft Z3 solver [6] with real arithmetic logic. Z3 found that the defined inequation system is unsatisfiable, pointing that the mechanism is incentive compatible.

3.3. Experimental validation

If previous defined methods cannot be used to test the incentive compatibility of a mechanism due to the complexity of the mechanism or to the impossibility of a solver to decide if the inequation system is or is not satisfiable, the incentive compatibility of the mechanism can be deduced from empirical experimentation [7]. A way of determining if a mechanism is incentive compatible or not is to compare the benefits that cheater agents obtain against the benefits they would have obtained by bidding truthfully. Results obtained show that bidders obtain 28,10% higher benefits when bidding truthfully than when cheating (average of 200 simulations). Another common experiment is to use learning agents [8]. We have implemented a reinforcement learning method so agents learn the probability of following one strategy or another (i.e. cheating or not). The experimental results obtained show that the agents learn with the highest probability (0.7) to bid truthfully. Detailed results can be found on paper [9].

4. Conclusions

In this work we have described three different ways to demonstrate the incentive compatibility of a multi-attribute mechanism. The first one shows that if a mechanism commits the exactness, participation, monotonicity and criticality conditions, it can be considered incentive compatible. The second one consisted in using a constraint solver to prove that there are no cases where a bidder can obtain higher utility by lying than by revealing its true values. Finally, an experimental validation is required for the cases where the first two demonstrations cannot be carried out due to the complexity of the mechanism. We have illustrated the three demonstration methods in a multi-attribute mechanism. The three demonstrations clearly showed that the mechanism is incentive compatible.Regarding the usability of the methods, the authors consider that the second methodology is the simplest and easiest way to ensure the strategyproof of an auction mechanism. However, some mechanisms may be too complex for solvers to determine the satisfiability of the inequation systems which define them, reducing the utility of this demonstration to a restricted number of mechanisms. When this occurs, demonstrations based on meeting sufficient conditions and empirical checking should be used.

References

[1] Y. Chen. Incentive-compatible mechanisms for pure public goods: A survey of experimental research. *Handbook of experimental economics results*, 1:625–643, 2008.

[2] A. Pla, B. López, and J. Murillo. Multi-attribute auction mechanism for supporting resource allocation in business process enactment. In *STAIRS*, volume 241 of *FAIA*, pages 228–239. IOS Press, 2012.

[3] Y. Che. Design competition through multidimensional auctions. *RAND J. of Economics*, 24:668–680, 1993.

[4] A. Pla, B. López, and J. Murillo. Multi criteria operators for multi-attribute auctions. In *Modeling Decisions for Artificial Intelligence*, Lecture Notes in Computer Science, pages 318–328. 2012.

[5] D. Lehmann, L.I. O'callaghan, and Y. Shoham. Truth revelation in approximately efficient combinatorial auctions. *J. ACM*, 49:577–602, September 2002.

[6] L. De Moura and N. Bjrner. Z3: An efficient SMT solver. *Tools and Algorithms for the Construction and Analysis of Systems*, 4963/2008:337–340, 2008.

[7] Jayson L Lusk and Ted C Schroeder. Are choice experiments incentive compatible? a test with quality differentiated beef steaks. *American Journal of Agricultural Economics*, 86(2):467–482, 2004.

[8] Vishnuteja Nanduri and Tapas K Das. A reinforcement learning model to assess market power under auction-based energy pricing. *Power Systems, IEEE Transactions on*, 22(1):85–95, 2007.

[9] Albert Pla and Beatriz Lpez and. Proving incentive compatibility in multi-attribute auctions. Research report IIiA 12-01-rr, Institute of Informatics and Applications, University of Girona, 2012.