

# NON LINEAR IDENTIFICATION OF UNDERWATER VEHICLES

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**Abstract:** This paper presents an identification method for non linear models of Unmanned Underwater Vehicles (UUV's). The proposed method operates both off-line and recursively and can be applied to a quite general class of non linear multivariable models. The validity of the method is demonstrated through an application to the identification of the uncoupled non linear surge-yaw-pitch dynamics of the URIS underwater robotic vehicle

**Keywords:** Identification, parameter estimation, numerical methods, unmanned underwater vehicles.

## 1. INTRODUCTION

The application of system identification techniques to marine vehicles is concerned with the estimation, on the basis of experimental measurements, of a number of parameters or of hydrodynamic derivatives that characterise the vehicle's dynamics [1]. Such measurements, collected during full-scale trials by the on-board sensors, are processed by a parameter estimation routine [6]. The identification methods proposed in the recent years for UUV identification generally operate off-line and the underlying mathematical models are of the scalar type. Furthermore, they are essentially deterministic, since the effects of disturbances affecting the UUV dynamics and of measurement noise are not taken into consideration [2, 3]. In [10] we applied the proposed method to the identification of GARBI UUV. In that case we used the experimental data obtained during a previous identification of GARBI [9]. The experimental set-up was complex so, only a few experimental data was available. In this paper, we applied the proposed method to URIS UUV. The lab set-up mounted for URIS identification is better, allowing us to carry out an exhaustive experimentation, so the quality of the model is improved. A concise review of the UUV mathematical models generally used in the literature is presented in section 2. In section 3 the

identification method is described in the off-line mode and a recursive identification algorithm is presented in subsection 3.1. In section 4 a description of URIS underwater robotic vehicle and the experimental set-up is reported and, in section 5, the identification results obtained from real experiments are shown. Finally some concluding remarks are done in section 6.

## 2. UUV MATHEMATICAL MODELS

The mathematical model of a wide class of UUVs can be expressed [5], with respect to a local body reference system, by a set of non linear coupled Newtonian equations of the form:

$$M(x,t)\dot{x} = f(x,t) + \tau(t) + g(x,t) \quad (1)$$

where  $x(t) \in R^6$  is the vehicle's state vector, generally constituted by linear and angular velocities, i.e.  $x = [u, v, w, p, q, r]^T$  consisting of surge, sway, heave, roll rate, pitch rate and yaw rate,  $M(x,t) \in R^{6 \times 6}$  is the body inertial matrix including hydrodynamic added masses,  $f(x,t) \in R^6$  is the vector of kinematic forces and moments,

$\tau(t) \in R^6$  is the vector of control forces and moments from thrusters and control surfaces,  $g(x,t) \in R^6$  is a vector including all the other hydrodynamic forces and moments.

Identification of the complete set of coefficients and hydrodynamic derivatives which appear in Equation (1) is a rather complex task, owing to the very high number of parameters, to nonlinearities and to space-time variant effects. The UUV considered in this paper, better described in Section 4, will be assumed to be adequately described by three decoupled motions, i.e. surge  $u$ , yaw  $r$  and pitch  $q$ . Such motions can be described, in a more compact form [9] by the following set of decoupled equations:

$$\dot{x}_i = \alpha_i x_i + \beta_i x_i |x_i| + \gamma_i \tau_i + \delta_i \quad i=1,3 \quad (2)$$

where the index values  $i=1$  corresponds to surge,  $i=2$  to yaw and  $i=3$  to pitch. The coefficients  $\alpha_i$  and  $\beta_i$  are the linear and quadratic drag coefficients,  $\gamma_i$  is the inverse of the diagonal element of the reduced order vehicle inertia matrix and  $\delta_i$  is a bias term that can take into account buoyancy effects as well as other exogenous forces or moments. The term  $\tau_i$  represents the active force or moment excited by thrusters.

### 3. IDENTIFICATION METHOD

As it can be easily recognised, the UUV decoupled dynamics, as expressed by Equation (2) is a particular case of a more general class of non linear system that are linear with respect to the system parameter vector:

$$\dot{x} = \phi(x(t), \tau(t))\theta \quad (3)$$

$$y_k = x(t_k) + e_k$$

where  $x = [x_1 \ x_2 \ x_3]^T$  is the state vector and  $\tau = [\tau_1 \ \tau_2 \ \tau_3]^T$  is the vector of active force and moments applied by the thrusters and  $\phi \in R^{12 \times 3}$  is a matrix valued function depending only on state and control vectors, while  $\theta \in R^{12}$  is a constant and unknown parameter vector that characterises the system dynamics.

$$\phi = \begin{bmatrix} x_1 & x_1|x_1| & \tau_1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & x_2 & x_2|x_2| & \tau_2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x_3 & x_3|x_3| & \tau_3 & 1 \end{bmatrix} \quad (4)$$

$$\theta = [\alpha_1 \ \beta_1 \ \gamma_1 \ \delta_1 \ \alpha_2 \ \beta_2 \ \gamma_2 \ \delta_2 \ \alpha_3 \ \beta_3 \ \gamma_3 \ \delta_3]^T \in R^{12}$$

A discrete-time measurement vector  $y_k$  corrupted by zero-mean gaussian noise  $e_k$  is assumed.

The identification problem consists of estimating the unknown parameter vector  $\theta$  on the basis of a finite number of discrete time measurements of input vectors  $\{\tau_k\}_{k=1}^N$  and output vectors  $\{y_k\}_{k=1}^N$ . According to the Output Prediction Error method [6], identification of parameter vector  $\theta$  is equivalent to the minimization of a scalar cost function of the form:

$$J(\theta) = \frac{1}{N} \sum_{k=1}^N \varepsilon_k^T \Lambda_k \varepsilon_k \quad (5)$$

The cost function is constituted by a weighted sum of squares of prediction errors  $\varepsilon(t_k)$ , which are the difference between the observed output vectors and the one-step prediction of the output  $\hat{y}_k$ , i.e.:

$$\varepsilon_k = y_k - \hat{y}_k \quad (6)$$

The positive definite matrices  $\{\Lambda_k\}_{k=1}^N$  consist of weights that should take into account the reliability of measurements at each discrete time instant.

It is worth noting that if the measurement noise vector  $e_k$  is zero-mean then:

$$\hat{y}_k = \hat{x}(t_k) \quad (7)$$

where  $\hat{x}(t_k)$  denotes the expected state vector at time  $t_k$ . In order to determine a solution to the minimization of the cost function expressed by Equation (5), it is necessary that an estimate of one-step output predicted output  $\hat{y}_k$  is available. For this purpose, let us formally integrate both sides of state equation in Equation (8) between two subsequent time instants  $t_{k-1}$  and  $t_k$ , obtaining:

$$x(t_k) - x(t_{k-1}) = \left[ \int_{t_{k-1}}^{t_k} \phi(x(s), \tau(s)) ds \right] \cdot \theta \quad (8)$$

If, taking into account Equation (11), it is assumed that  $x(t_{k-1}) = \tilde{y}_{k-1}$  where  $\tilde{y}_{k-1}$  is a properly filtered version of the output vector  $y_{k-1}$ , i.e. if we assign to the unknown state vector a corresponding filtered output, then we obtain the following estimate for the state vector at time  $t_k$ :

$$\hat{x}(t_k) = \tilde{y}_{k-1} + F_k \theta \quad (9)$$

where

$$F_k = \int_{t_{k-1}}^{t_k} \phi(\hat{x}(s), \tau(s)) ds \quad (10)$$

and thus, we can achieve an evaluation of the one-step prediction error of Equation (10) in the form:

$$\varepsilon_k = \tilde{y}_k - \tilde{y}_{k-1} - F_k \theta \quad (11)$$

By inserting this evaluation of the one-step prediction error into the cost function expression of Equation (4), it is finally possible to find the value of the parameter vector  $\theta_{LS}(N)$  that minimizes the cost function on the basis of  $N$  observations. In fact, it can be easily demonstrated [5] that, under a regularity condition of the matrix appearing in the normal equation, such problem admits a unique solution obtained through the Least Squares (LS) algorithm:

$$\theta_{LS}(N) = (F^T(N) \cdot \Lambda(N) \cdot F(N))^{-1} \cdot \quad (12)$$

$$F^T(N) \cdot \Lambda(N) \cdot Y(N)$$

where  $\Lambda(N)$ ,  $F(N)$ ,  $Y(N)$  are:

$$\Lambda(N) = \begin{bmatrix} \Lambda_1 & 0 & \dots & 0 \\ 0 & \Lambda_2 & \dots & 0 \\ \cdot & \cdot & \dots & \cdot \\ 0 & 0 & \dots & \Lambda_N \end{bmatrix} \quad (13)$$

$$F(N) = \begin{bmatrix} F_1 \\ F_2 \\ \cdot \\ \cdot \\ F_N \end{bmatrix}, \quad Y(N) = \begin{bmatrix} \tilde{y}_1 - \tilde{y}_0 \\ \tilde{y}_2 - \tilde{y}_1 \\ \cdot \\ \cdot \\ \tilde{y}_N - \tilde{y}_{N-1} \end{bmatrix} \quad (14)$$

The elements of the compound matrix  $F(N)$  can be obtained by using a numerical integration algorithm [8] that takes into account the form of the filtered output vector.

### 3.1. Recursive identification

The above presented identification method operates off-line, in the sense that the parameter estimation is done by collecting a long enough record of input-output data and then processing them in one shot. If the identified model has to be used for prediction or control, however, this procedure may be not adequate. Furthermore, the method presumes that the parameter vector is time invariant, but in the case of an underwater vehicle this assumption is generally quite unrealistic, as far as a relatively long time horizon is considered. It is possible to extend the above identification algorithm to a recursive mode, in such a way that also slowly time variant parameter can be recursively estimated. The basic idea is to compute the new parameter vector estimate  $\hat{\theta}(k)$  at time  $k$  by adding some correction vector to the

previous parameter estimate  $\hat{\theta}(k-1)$ . It can be demonstrated that the following Recursive Least Squares (RLS) estimate can be deduced.

$$\hat{\theta}(k) = \hat{\theta}(k-1) + \Gamma_k (Y_k - F_k \hat{\theta}(k-1))$$

$$\Gamma_k = P_{k-1} F_k^T (\lambda I + F_k P_{k-1} F_k^T)^{-1} \quad (15)$$

$$P_k = \frac{1}{\lambda} (I - \Gamma_k F_k) P_{k-1}$$

where

$$P_k \in R^{12 \times 12}, \Gamma_k \in R^{12 \times 3}, F_k \in R^{3 \times 12}, Y_k \in R^{3 \times 12}$$

The RLS algorithm has been deduced under the assumption that the weighting matrices have the form

$$\Lambda_k = \lambda_{n-k} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (16)$$

The algorithm generally converges to the true parameter vector. As remarked [7], the RLS algorithm is capable to take into account time-invariant processes as well as non-stationary systems. The real parameter  $\lambda \in (0, 1]$ , often called forgetting factor, takes into account that most recent data are weighted more than old ones. The adjustment of  $\lambda$  is generally a tradeoff between high robustness against disturbances (large  $\lambda$ ) and tracking capability (small  $\lambda$ ). The forgetting factor is generally set between 0.9 and 1. The algorithm RLS requires an initial parameter estimate  $\hat{\theta}(0)$  and an initial assignment of the positive definite matrix  $P_0$ , that generally should reflect the degree of a-priori knowledge about unknown parameter vector [6], [7].

## 4. DESCRIPTION OF URIS UUV AND EXPERIMENTAL SET-UP

URIS robot was developed at the University of Girona with the aim of building a small-sized AUV. The hull is composed of a stainless steel sphere with a diameter of 350mm, designed to withstand pressures of 3 atmospheres (30 meters depth). On the outside of the sphere there are two video cameras (forward and down looking) and 4 thrusters (2 in X direction and 2 in Z direction). Due to the stability of the vehicle in pitch and roll, the robot has four degrees of freedom (DOF); surge, sway, heave and yaw. Except for the sway DOF, the others DOFs can be directly controlled. The robot has an onboard PC-104 computer, running the real-time operative system QNX. In this computer, the low and high level controllers are executed. An umbilical wire is used

for communication, power and video signal transmissions. The navigation system [4] is currently being executed on an external computer. All the experiments were carried out in a water tank located in our lab (see Fig. 2).

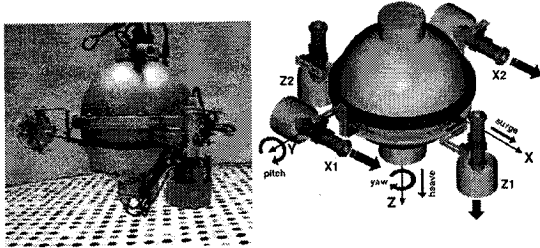


Fig. 1. (left) URIS in the water tank. (right) URIS reference frame

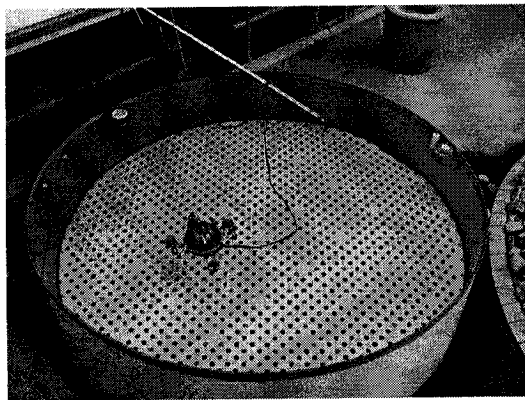


Fig. 2. Water tank used in the identification experiments.

## 5. IDENTIFICATION

### 5.1. Methodology

The identification method has been tested with data measured with URIS UUV during experiments where the three motions (surge, pitch and yaw) were excited separately. The unique variables measured were the propeller angular speed and the robot position. Force/torque was computed using the thruster model and speed was computed through numerical differentiation.

### 5.2. Identification results for the yaw DOF

Six different trials were run for the yaw DOF, 4 using step signals and 2 using PBRs signals. Four of them (3 steps and 1 PBRs) were validated and 2 were discarded. In all cases, no significant improvement was observed taking into account the quadratic damping. For this reason, it was considered to be zero. This is usual for very low speed robots like the one considered here. Table 1 shows the results of the validated experiments as well as their average. The estimated parameters are shown together with their standard deviation and the cost of the whole experiment.

Table 1 Parameter results for yaw DOF

Exp		$\alpha_2$	$\gamma_2$	$\delta_2$	$J_2$
1	$\theta$	1.1865	0.6261	-0.017	5.7759e-4
	$\sigma$	0.0031	0.0015	0.0006	
2	$\theta$	1.3449	0.6178	0.1076	6.9684e-4
	$\sigma$	0.0053	0.0024	0.0010	
3	$\theta$	1.1635	0.4221	-0.255	8.7854e-4
	$\sigma$	0.0029	0.0010	0.0011	
4	$\theta$	1.2326	0.3468	0.1268	0.0020
	$\sigma$	0.0053	0.0014	0.0021	
Mean	$\theta$	1.2426	0.5173	-0.050	8.38e-04
	$\sigma$	0.00355	0.00145	0.001	

Lets us consider in the following paragraphs, experiment 3 as a case of study to illustrate the procedure. Fig. 3 shows the input signals (force and the filtered force, acceleration, speed and position) of the experiment (note that acceleration is shown only for clarity, it is not used in the identification). The statistical validation of the results is reported in Fig. 4. It shows, from the top to the bottom, the residuals, their histogram and the normalized autocorrelation function. The residuals are clearly gaussian and zero-mean and after 0.5 seconds lag, the residuals are clearly within the 95% confidence region. Hence, results can be considered statistically good. The performance of the model is presented in Fig. 5, where the measured velocity is compared with the simulated velocity. The top graphic, reports the measured velocity compared with the one step predicted velocity evaluated in working point of the previous measured velocity. The bottom graphic shows the measured velocity compared with the one simulated by the model. In both cases, a very good agreement can be observed. Finally, Fig.5 and Fig.6 show the long term simulation capability of the model with respect to the measured values, for yaw velocity and yaw angle.

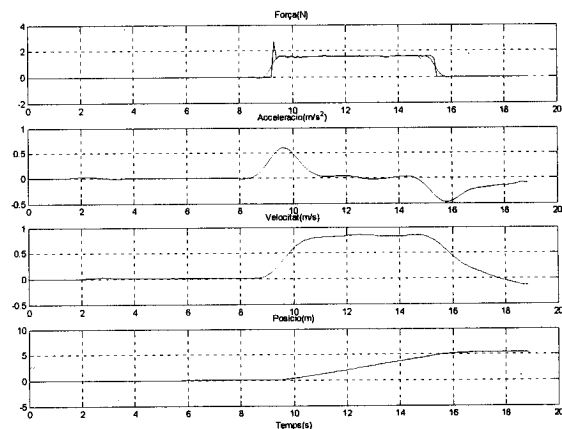


Fig. 3. Input signals for yaw DOF (experiment 3)

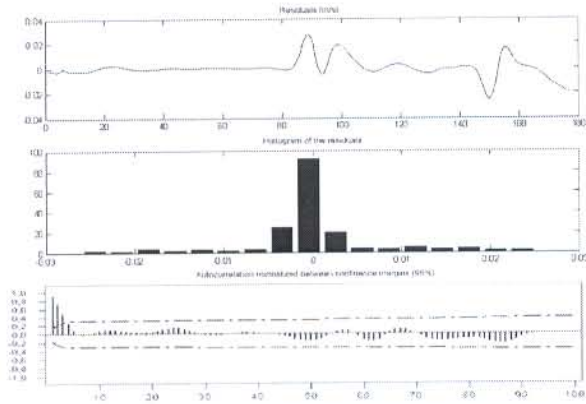


Fig. 4. Statistical validation for yaw DOF

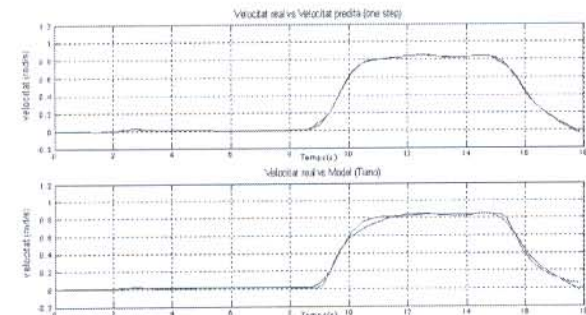


Fig. 5. Speed response for yaw DOF

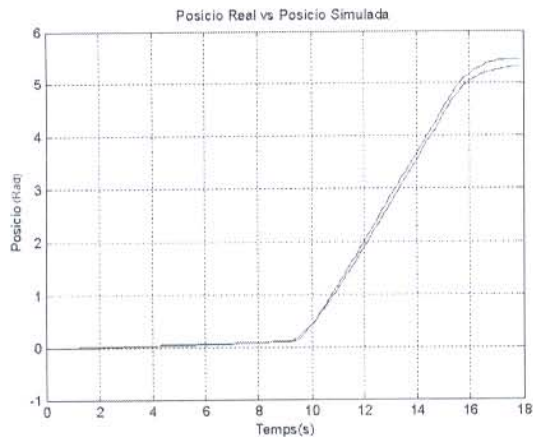


Fig. 6. Position response for yaw DOF

### 5.3. Identification results for surge and pitch

Table 2 show analogous results obtained for surge DOF validated experiments, while Fig.7 and 8 show the performance of the model. The same is reported for pitch in table 3, Fig.9 and 10. Finally, the chosen model after averaging the validated experiments is shown in Table 5.

Table 2 Parameter results for surge DOF

Exp		$\alpha_1$	$\gamma_1$	$\delta_1$	$J_1$
1	$\theta$	0.3243	0.0178	-0.0018	1.630e-4
	$\sigma$	0.0023	0.0001	0.0002	
2	$\theta$	0.3185	0.0201	0.0041	1.895e-4
	$\sigma$	0.0016	0.0001	0.0001	
3	$\theta$	0.3237	0.0173	0.0013	1.892e-4
	$\sigma$	0.0016	0.0001	0.0002	
Mean	$\theta$	0.3222	0.0184	0.0012	1.805e-4
	$\sigma$	0.0018	0.0001	0.00016	

Table 3 Parameter results for pitch DOF

Exp		$\alpha_3$	$\gamma_3$	$J_3$
1	$\theta$	0.6486	1.0175	6.4228e-4
	$\sigma$	0.0013	0.0007	
2	$\theta$	0.6586	0.8697	4.3642e-4
	$\sigma$	0.0009	0.0004	
3	$\theta$	0.6895	1.0735	5.3489e-4
	$\sigma$	0.0019	0.0011	
Mean	Param	0.66395	0.9885	5.5032e-4
	$\sigma$	0.0011	0.0006	

Table 4 Summary of results

DOF	$\alpha$	$\beta$	$\gamma$	$\delta$	$J$
Surge	0,3302	0	0,0169	-0,0070	1.572e-4
Pitch	0.6514	0	0.8052	0	3.764e-4
Yaw	1.1006	0	0.5236	-0.0435	1.038e-4

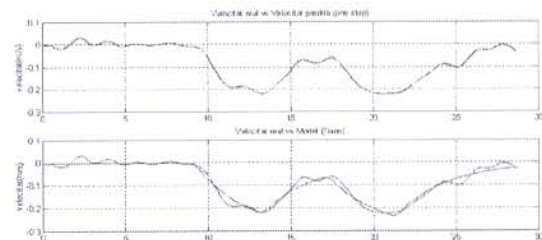


Fig. 7. Speed response for surge DOF

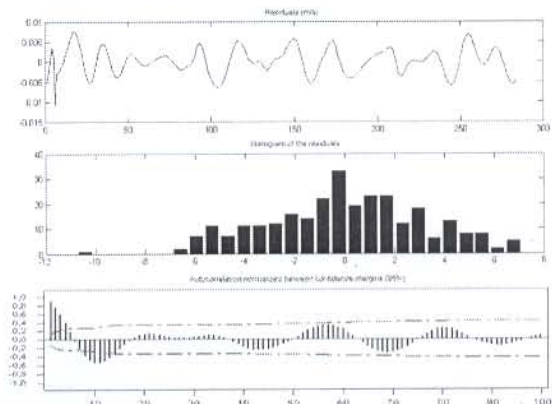


Fig. 8. Statistical validation for surge DOF

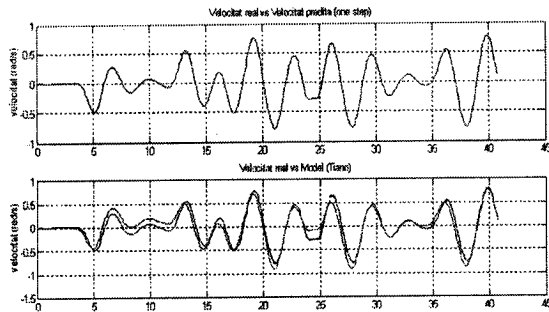


Fig. 9. Speed response for pitch DOF

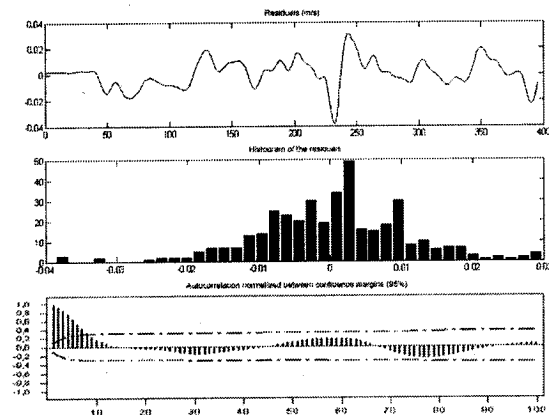


Fig. 10. Statistical validation for pitch DOF

#### 5.4. Recursive identification results

The recursive identification algorithm has also been tested on the basis of simulated data, obtained from the model identified off-line. In this case the parameter to be estimated is constituted by the vector

$$\theta = [\alpha_1 \ \gamma_1 \ \delta_1 \ \alpha_2 \ \gamma_2 \ \delta_2 \ \alpha_3 \ \gamma_3]^T \in \mathbb{R}^8$$

and the data were generated through a simulated model.

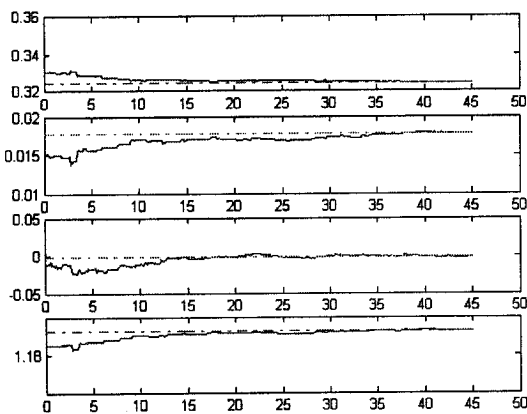


Fig.11. Convergence of RLS algorithm

The results obtained by the RLS algorithm are shown in Fig. 11, where the convergence pattern of the first four components of the estimated parameter vector to the true one is illustrated.

## 6. CONCLUSIONS

An identification method for a wide class of non linear systems has been presented. The method, operating off-line, has been applied to the identification of URIS UUV on the basis of real data. The results obtained indicate that the method can achieve excellent numerical performance. The identified model has proven to be statistically good and will be used in the near future for simulation and control.

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