# ON THE IDENTIFICATION OF NON LINEAR MODELS OF UNMANNED UNDERWATER VEHICLES

**M.Carreras*, A.Tiano†, A. El-Fakdi*, A. Zirilli†, P.Ridao***

[*] *Institut of Informatics and Applications*
*University of Girona, Spain*

[†] *Department of Information and Systems*
*University of Pavia, Italy*

Abstract: *This paper presents an identification method for the off-line identification of non linear models of Unmanned Underwater Vehicles (UUV's). The proposed method can be applied to a quite general class of non linear multivariable models and is characterised by an excellent numerical performance. The validity of the method is demonstrated through an application to the identification of the dynamic behaviour of the URIS underwater robotic vehicle.*


Keywords: Identification, parameter estimation, numerical methods, unmanned underwater vehicles.

## 1. INTRODUCTION

The application of system identification techniques to naval vehicles is concerned with the estimation, on the basis of experimental measurements, of a number of parameters or of hydrodynamic derivatives that characterise the vehicle's dynamics (Abkowitz, 1980). Such measurements, collected during full-scale trials by the on-board sensors, are processed by a parameter estimation routine (Ljung, 1987).
The identification methods that, in the recent years, have been proposed for UUV identification generally operate off-line and the underlying mathematical models are of the scalar type. Furthermore, they are essentially deterministic, since the effects of disturbances affecting the UUV dynamics and of measurement noise are not taken into consideration (Caccia, 2000). An on-line deterministic identification method that has been most recently proposed (Smallwood, 2001) is limited to scalar decoupled models.
In (Tiano, 2002) we applied the proposed method to the identification of GARBI UUV. In that case we used the experimental data obtained during a previous identification of GARBI (Ridao, 2001). The experimental set-up was complex so, only a few experimental data was available. In this paper, we applied the proposed method to URIS UUV. The lab set-up mounted for URIS identification is better, allowing us to carry out an exhaustive experimentation, so the quality of the model is improved.
A brief review of the UUV mathematical models generally used in the literature is presented in sub-section 1.1 of this paper. In section 2 a general method is presented for the off-line identification of a wide class of non linear continuous-time systems. Some of the numerical aspects of the method are discussed in sub-section 2.1. In section 3 a description

of URIS underwater robotic vehicle and our experimental set-up is reported and, in section 4, the identification results obtained for the identification of URIS are shown. Finally some concluding remarks are done in section 5.

### *1.1 UUV Mathematical models*

An UUV can be modelled using the basic physical laws governing the dynamic behaviour of a system. As described in the literature (Fossen, 1994), the non-linear hydrodynamic equation of motion of an underwater vehicle with 6 DOF, in the body fixed frame, can be conveniently expressed as:

$$^{B}t + G(h) - D\left(^{B}u\right)\, ^{B}u + t_{p} = \left(^{B}M_{RB} + M_{A}\right)\cdot\, ^{B}\dot{u} + \left(^{B}C_{RB}(^{B}u) + C_{A}\left(^{B}u\right)\right)\cdot\, ^{B}u \qquad (1)$$

$$^{B}t = Bu \qquad (2)$$

$$u_i = |w_i|w_i \qquad (3)$$

$$t_i = C_{Ti}\,|w_i|w_i \qquad (4)$$

The resultant force and moment exerted by the thrusters $^{B}t$ is obtained from (2) and (3), while equation (4) provides the force exerted by thruster *i*, see (Newman, 1989) for details.

Identification of the complete set of coefficients and hydrodynamic derivatives which appear in Equation (1) is a rather complex task, owing to the very high number of parameters, to nonlinearities and to space-time variant effects. The identification problem can be much more easily approached if the following simplifications apply:

- *$D(^{B}n)$* consists of the lineal and quadratic damping forces and can be assumed diagonal.
- $^{B}M_{RB}$ and $^{B}M_{A}$ can be assumed diagonal (this is true for URIS UUV due to its spherical shape see 3).
- the body frame is located at the gravity centre

Moreover, if the robot is actuated in a single DOF during the identification experiments, further simplifications can be carried out. Let us consider, for instance, the dynamic equation for the yaw DOF:

$$N + (-x_b\, cq\, sf\, B - y_b\, sq\, B) - (N_r + N_{r|r|}\,|r|)\cdot r + t_p = (I_Z - N_{\dot{r}})\dot{r} + (mv - Y_{\dot{V}}\, v - mu + X_{\dot{U}}\, u - M_{\dot{q}}\, q - I_X\, p + K_{\dot{p}}\, p)\cdot r \qquad (5)$$

which follows the standard notation proposed in (Fossen, 1994). If we excite the robot in a single DOF, yaw in this case, in such a way that:

- *$r^{1}0$ and $u=v=w=p=q=0$*
- *$q=f=0$*

then we run an uncoupled experiment so, equation (5) becomes:

$$\dot{r} = \underbrace{\frac{N}{(I_Z - N_{\dot{r}})}}_{g} - \underbrace{\frac{N_r}{(I_Z - N_{\dot{r}})}}_{a}\cdot r - \underbrace{\frac{N_{r|r|}\,|r|}{(I_Z - N_{\dot{r}})}}_{b}\cdot r + \underbrace{\frac{t_p}{(I_Z - N_{\dot{r}})}}_{d} \qquad (6)$$

The same procedure can be applied to each degree of freedom so, we can consider a generic uncoupled equation of motion for degree *i* as:

$$\dot{x}_i = a_i v_i +_i x_i\,|x_i| + g_i t_i + d_i \qquad (7)$$

where the state variable $x$ represents speed in our model. Hence, things become more easy if we use equation (7) for the identification. It is worth to note that the terms of equation (5) not present in equation (6), will be identified by running an uncoupled experiment of the corresponding degree of freedom. Hence, if we run *n* experiments (*n* is the number of DOFs) we can get the complete model.

## 2. IDENTIFICATION METHOD

As it can be easily recognised, the UUV decoupled dynamics, as expressed by Equation (7) is a particular case of a more general class of non linear system that are linear with respect to the system parameter vector. If it is assumed that the state vector $x \in R^n$ is completely controllable by the control vector $t \in R^m$ and completely observable at discrete time instants $\{t_k\}_{k\geq 0}$ through the output vector $y(t_k) \in R^n$, corrupted by the additive zero-mean noise vector $e(t_k) \in R^n$, the system dynamics can be expressed by:

$$\begin{aligned} \dot{x} &= f(x(t), t(t))q \\ y(t_k) &= x(t_k) + e(t_k) \end{aligned} \qquad (8)$$

where $f \in R^{nxl}$ is a matrix valued function depending only on state and control vectors, while $q \in R^l$ is a constant and unknown parameter vector that characterises the system dynamics.

The identification problem consists of estimating the unknown parameter vector $q$ on the basis of a finite number of discrete time measurements of input vectors $\{t_k\}_{k=1}^{N}$ and output vectors $\{y(t_k)\}_{k=1}^{N}$.

According to the Output Prediction Error method (Ljung, 1987), identification of parameter vector $q$ is equivalent to the minimization of a scalar cost function of the form:

$$J(q) = \frac{1}{N}\sum_{k=1}^{N} e^{T}(t_k)W^{-1}(t_k)e(t_k) \qquad (9)$$

The cost function is constituted by a weighted sum of squares of prediction errors $e(t_k)$, which are the difference between the observed output vectors and the one-step prediction of the output $\hat{y}(t_k)$, i.e.:

$$e(t_k) = y(t_k) - \hat{y}(t_k) \qquad (10)$$

The positive definite matrices $\{W^{-1}(t_k)\}_{k=1}^{N}$ consist of weights that should take into account the reliability of measurements at each discrete time instant. For example, if the noise vector sequence, $\{e(t_k)\}_{k=1}^{N}$ is the realisation of a zero-mean stochastic process with uncorrelated components and known finite second order moments, a “reasonable” choice of the weighting matrix sequence could consist of the diagonal matrix of the individual component variances.

It is worth noting that if the measurement noise vector $e(t_k)$ is zero-mean then:

$$\hat{y}(t_k) = \hat{x}(t_k) \qquad (11)$$

where $\hat{x}(t_k)$ denotes the expected state vector at time $t_k$.

In order to determine a solution to the minimization of the cost function expressed by Equation (9), it is necessary that an estimate of one-step output predicted output $\hat{y}(t_k)$ is available. For this purpose, let us formally integrate both sides of state equation in Equation (8) between two subsequent time instants $t_{k-1}$ and $t_k$, obtaining:

$$x(t_k) - x(t_{k-1}) = \left[ \int_{t_{k-1}}^{t_k} f(x(s), t(s))ds \right] \cdot q \qquad (12)$$

If, taking into account Equation (11), it is assumed that $x(t_{k-1}) = \tilde{y}(t_{k-1})$, where $\tilde{y}(t_{k-1})$ is a properly filtered version of the output vector $y(t_{k-1})$, i.e. if we assign to the unknown state vector a corresponding filtered output, then we obtain the following estimate for the state vector at time $t_k$:

$$\hat{x}(t_k) = \tilde{y}(t_{k-1}) + F_k \cdot q \qquad (13)$$

where

$$F_k = \int_{t_{k-1}}^{t_k} f(\hat{x}(s), t(s))ds \qquad (14)$$

and thus, we can achieve an evaluation of the one-step prediction error of Equation (10) in the form:

$$e(t_k) = \tilde{y}(t_k) - \tilde{y}(t_{k-1}) - F_k \cdot q \qquad (15)$$

By inserting this evaluation of the one-step prediction error into the cost function expression of Equation (9), it is finally possible to find the value of the parameter vector $q_{LS}(N)$ that minimizes the cost function on the basis of $N$ observations. In fact, it can be easily demonstrated (Lijung, 1994) that, under a regularity condition of the matrix appearing in the normal equation, such problem admits a unique solution obtained through the Least Squares (LS) algorithm :

$$q_{LS}(N) = (F^T(N) \cdot W^{-1}(N) \cdot F(N))^{-1} \cdot F^T(N) \cdot W^{-1}(N) \cdot Y(N) \qquad (16)$$

where $W(N) \in R^{nNxnN}$ , $F(N) \in R^{nNxl}$, $Y(N) \in R^{nNx1}$ and

$$W(N) = \begin{bmatrix} W(t_1) & 0 & ...... & 0 \\ 0 & W(t_2) & ...... & 0 \\ . & . & ...... & . \\ 0 & 0 & ...... & W(t_N). \end{bmatrix} \qquad (17)$$

$$F(N) = \begin{bmatrix} F_1 \\ F_2 \\ . \\ . \\ F_N \end{bmatrix}, \; Y(N) = \begin{bmatrix} \tilde{y}(t_1) - \tilde{y}(t_0) \\ \tilde{y}(t_2) - \tilde{y}(t_1) \\ . \\ . \\ \tilde{y}(t_N) - \tilde{y}(t_{N-1}) \end{bmatrix} \qquad (18)$$

The elements of the compound matrix $F(N)$ can be obtained by using a numerical integration algorithm (Press, 1992) that takes into account the form of the filtered output vector.

### *2.1 Numerical aspects of the LS algorithm*

The main improvements of the proposed LS identification algorithm with respect to other apparently analogous techniques previously proposed like (Caccia, 2000) or (Smallwood, 2001) derive from its better numerical performance. The proposed algorithm, in fact, transforms the non linear differential Equation (8) into an integral equation expressed by Equation (12) and then the integrals expressed by Equation (14) are approximated by using the measured output data via a suitable numerical integration routine. If the integration routine, the integration step and the weighting matrix $W$ of Equation (14) are properly chosen, it has been demonstrated that the proposed method exhibits also a better robustness with respect to measurement errors. Another important practical improvement derives from the reduction of the measurement hardware, since, by the proposed method, there is no need to use acceleration sensors.

## 3. DESCRIPTION OF URIS UUV AND THE EXPERIMENTAL SET-UP

URIS robot was developed at the University of Girona with the aim of building a small-sized UUV. The hull is composed of a stainless steel sphere with a diameter of 350mm, designed to withstand pressures of 3 atmospheres (30 meters depth). On the outside of the sphere there are two video cameras (forward and down looking) and 4 thrusters (2 in X direction and 2 in Z direction). Due to the stability of the vehicle in pitch and roll, the robot has four degrees of freedom (DOF): surge, sway, heave and yaw. Except for the sway DOF, the others DOFs can be directly controlled. The robot has an onboard PC-104 computer, running the real-time operative system QNX. In this computer, the low and high level controllers are executed. An umbilical wire is used for communication, power and video signal transmissions. The navigation system (Carreras, 2003) is currently being executed on an external computer. All the experiments were carried out in a water tank located in our lab (see Fig. 2).

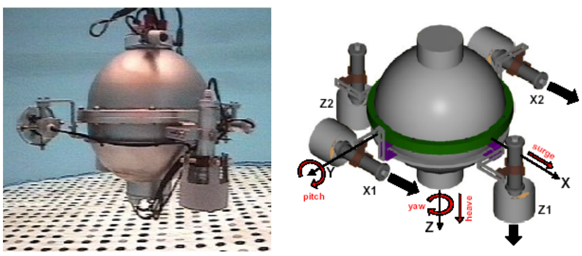


Fig. 1. (left) URIS in the water tank. (right) URIS reference frame

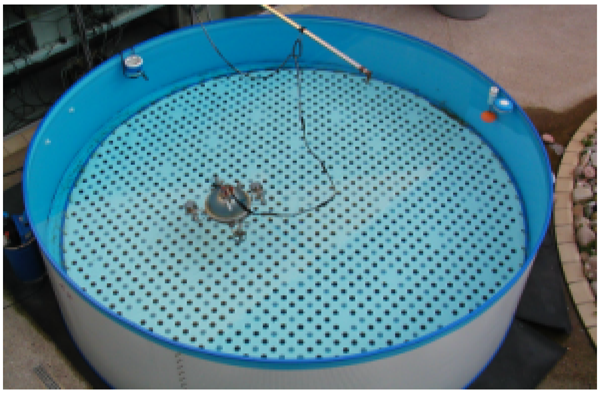

Fig. 2. Water tank used in the identification experiments.

# 4. IDENTIFICATION

### 4.1 Methodology

The identification method has been tested with data measured with URIS UUV during experiments where the three motions (surge, pitch and yaw) were excited separately. The unique variables measured were the propeller angular speed and the robot position. Force/torque was computed using the thruster model and speed was computed through numerical differentiation. The identification process was organised as follows:

*Phase 1: Uncoupled experiments*. The experiments excite the vehicle in one DOF, the input signals used were STEPs and PRBSs signals.

*Phase 2: Data validation*. The goal of this phase is to discard bad experiments, or part of them, like for instance when the force exerted by the wire cannot be neglected.

*Phase 3: Filtering*. To reduce the effects of measurement noise, variables were filtered. Position and force were filtered using Hamming filter and velocity was computed through position differentiation by means of a Savitzky-Golay filter.

*Phase 4: Off-line identification*. Equation (7) was used for estimating the model parameters as well as its standard deviation.

*Phase 5: Statistical validation*. The residuals of the model (so the one step prediction error) are statistically analysed. They should be gaussian, zero-mean and not autocorrelated.

*Phase 6: Test selection & Mean values*. From the previous phase, outliers in the experiments are detected and discarded. The results of the best ones are averaged to get the final values of the estimation.

*Phase 7: Simulation*. Finally, the identified model is used to simulate a true experiment. Real and simulated results, when both real system and model are subject to the same initial conditions and control inputs, are plotted for comparison.

### 4.2 Thruster identification

The static thruster model described in equation (4) is identified. A more complex dynamical model is not necessary since they are controlled on speed and their dynamics is much more faster than the robot dynamics. Thrust coefficients for horizontal and vertical thrusters are computed for both directions (see table 1). For details referring to the procedure for this identification see (Ridao, 2001).

Table 1 Thrust coefficients

| $C_T$ [N/rpm$^2$] | Positive sense | Negative sense |
|---|---|---|
| Horitzontal Thursters | $1.43 \cdot 10^{-5}$ | $1.48 \cdot 10^{-5}$ |
| Vertical Thrusters | $1.29 \cdot 10^{-5}$ | $1.25 \cdot 10^{-5}$ |

### 4.3 Identification results for the yaw DOF

Six different trials were run for the yaw DOF, 4 using step signals and 2 using PBRS signals. Four of them (3 steps and 1 PBRS) where validated and 2 were discarded. In all cases, no significant improvement was observed taking into account the quadratic damping. For this reason, it was considered to be zero. This is usual for very low speed robots like the one considered here. Table 2 shows the results of the validated experiments as well as their average. The estimated parameters are shown together with their standard deviation and the cost of the whole experiment.

Table 2 Parameter results for yaw DOF

| Exp | | $a_y$ | $g_y$ | $d_y$ | $J_y$ |
|---|---|---|---|---|---|
| 1 | θ | 1.1865 | 0.6261 | -0.017 | 5.7759e-4 |
| | σ | 0.0031 | 0.0015 | 0.0006 | |
| 2 | θ | 1.3449 | 0.6178 | 0.1076 | 6.9684e-4 |
| | σ | 0.0053 | 0.0024 | 0.0010 | |
| 3 | θ | 1.1635 | 0.4221 | -0.255 | 8.7854e-4 |
| | σ | 0.0029 | 0.0010 | 0.0011 | |
| 4 | θ | 1.2326 | 0.3468 | 0.1268 | 0.0020 |
| | σ | 0.0053 | 0.0014 | 0.0021 | |
| Mean | θ | 1.2426 | 0.5173 | -0.050 | 8.38e-04 |
| | σ | 0.00355 | 0.00145 | 0.001 | |

Note that physical parameters of the vehicle can be easily computed from those shown in table 2 by applying equation (6). Lets us consider in the following paragraphs, experiment 3 as a case of study to illustrate the procedure. Fig. 3 shows the input signals (force and the filtered force, acceleration, speed and position) of the experiment (note that acceleration is shown only for clarity, it is not used in the identification). The statistical validation of the results is reported in Fig. 4. It shows, from the top to the bottom, the residuals, their histogram and the normalized autocorrelation function. The residuals are clearly gaussian and zero-mean and after 0.5 seconds lag, the residuals are clearly within the 95% confidence region. Hence, results can be considered statistically good. The performance of the model is presented in Fig. 5, where the measured velocity is compared with the simulated velocity. The top graphic, reports the measured velocity compared with the one step predicted velocity evaluated in the working point of the previous measured velocity. The bottom graphic shows the measured velocity compared with the one simulated by the model. In both cases, a very good agreement can be observed. Finally, Fig.5 and Fig.6 show the long term simulation capability of the model with respect to the measured values, for yaw velocity and yaw angle.

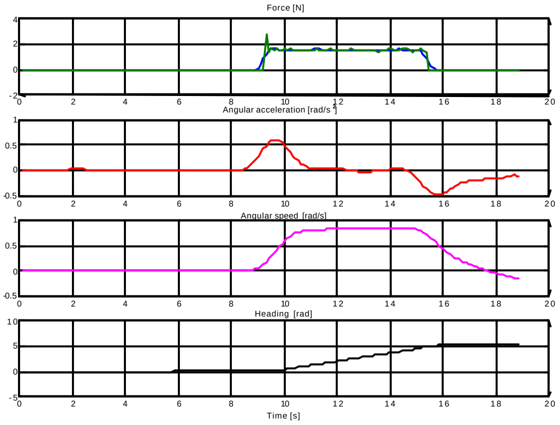


Fig. 3. Input signals for yaw DOF (experiment 3)

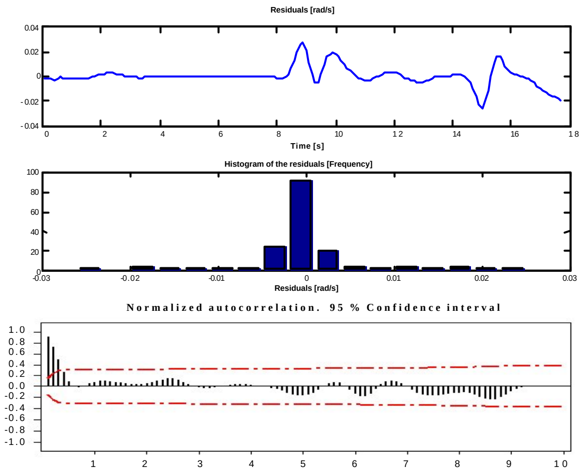


Fig. 4. Statistical validation for yaw DOF

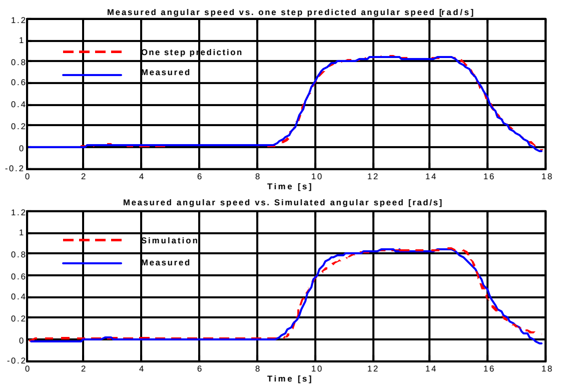


Fig. 5. Speed response for yaw DOF

*4.3 Identification results for the other DOFs*

Table 3 show analogous results obtained for surge DOF validated experiments, while Fig.7 and 8 show the performance of the model. The same is reported for pitch in table 4, Fig.9 and 10. Finally, the chosen model after averaging the validated experiments is shown in Table 5.

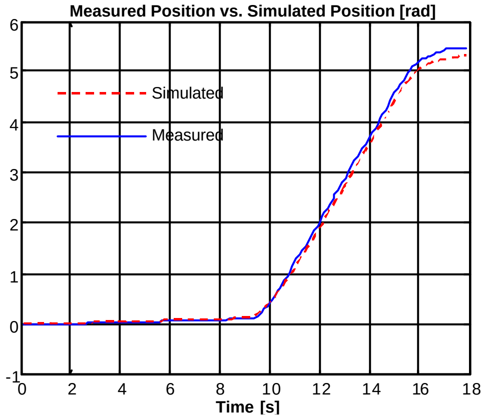


Fig. 6. Position response for yaw DOF

Table 3 Parameter results for surge DOF

| Exp | | $\alpha_x$ | $\gamma_x$ | $\delta_x$ | $J_x$ |
|---|---|---|---|---|---|
| 1 | θ | 0.3243 | 0.0178 | -0.0018 | 1.630e-4 |
| | σ | 0.0023 | 0.0001 | 0.0002 | |
| 2 | θ | 0.3185 | 0.0201 | 0.0041 | 1.895e-4 |
| | σ | 0.0016 | 0.0001 | 0.0001 | |
| 3 | θ | 0.3237 | 0.0173 | 0.0013 | 1.892e-4 |
| | σ | 0.0016 | 0.0001 | 0.0002 | |
| Mean | θ | 0.3222 | 0.0184 | 0.0012 | 1.805e-4 |
| | σ | 0.0018 | 0.0001 | 0.00016 | |

Table 4 Parameter results for pitch DOF

| Exp | | $a_\theta$ | $g_\theta$ | $J_\theta$ |
|---|---|---|---|---|
| 1 | θ | 0.6486 | 1.0175 | 6.4228e-4 |
| | σ | 0.0013 | 0.0007 | |
| 2 | θ | 0.6586 | 0.8697 | 4.3642e-4 |
| | σ | 0.0009 | 0.0004 | |
| 3 | θ | 0.6895 | 1.0735 | 5.3489e-4 |
| | σ | 0.0019 | 0.0011 | |
| Mean | θ | 0.66395 | 0.9885 | 5.5032e-4 |
| | σ | 0.0011 | 0.0006 | |

Table 5 Summary of results

| DOF | α | β | γ | δ | J |
|---|---|---|---|---|---|
| Surge | 0,3302 | 0 | 0,0169 | -0,0070 | 1.572e-4 |
| Pitch | 0.6514 | 0 | 0.8052 | 0 | 3.764e-4 |
| Yaw | 1.1006 | 0 | 0.5236 | -0.0435 | 1.038e-4 |

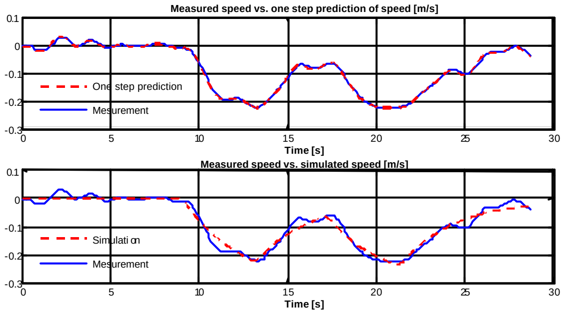


Fig. 7. Speed response for surge DOF

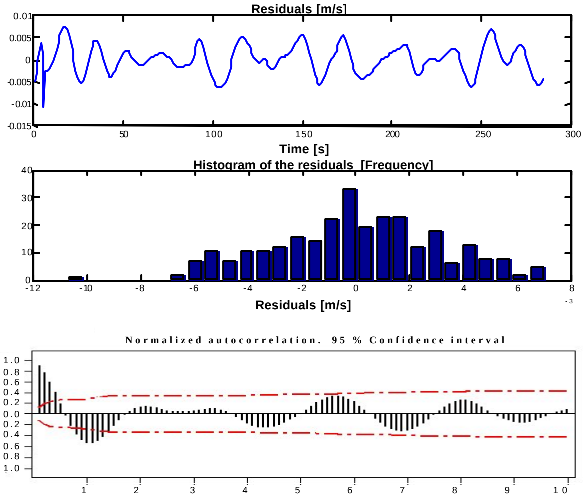


Fig. 8. Statistical validation for surge DOF

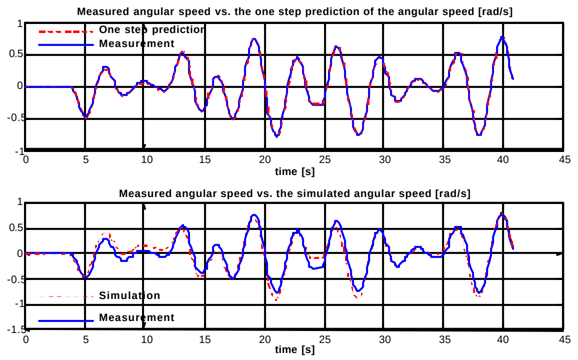


Fig. 9. Speed response for pitch DOF

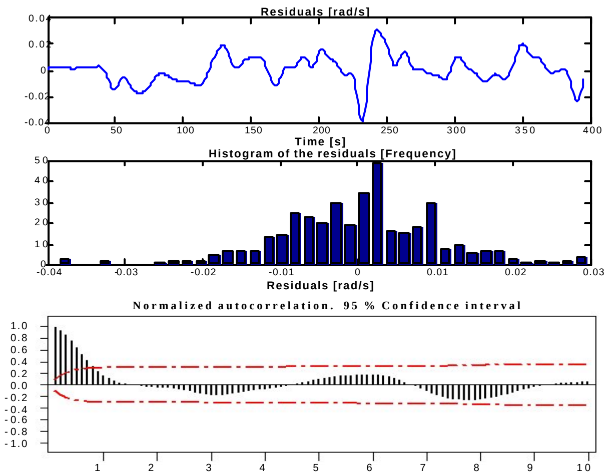


Fig. 10. Statistical validation for pitch DOF

## 5 CONCLUSIONS

An identification method for a wide class of non linear systems has been presented. The method, operating off-line, has been applied to the identification of URIS UUV on the basis of real data. The results obtained indicate that the method can achieve excellent numerical performance. The identified model has proven to be statistically good and will be used in the near future for simulation and control.

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